



# Quasiparticles in Leptogenesis

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# Outline

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  - May you put in thermal masses “by hand”?
  - Fermionic quasiparticles
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# Matter-Antimatter Asymmetry

- The universe today consists of matter and of practically no antimatter. Naively expected relic baryon density (annihilation into pions):  $\frac{n_B}{n_\gamma} = \frac{n_{\bar{B}}}{n_\gamma} \sim 10^{-20}$
- Observed baryon asymmetry:  $\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} \sim 10^{-9}$   
⇒ Huge number! Explanation?
- A matter-antimatter asymmetry can be dynamically generated if Sakharov's conditions [1967] are fulfilled:
  - baryon number violation
  - C and CP violation (reaction rates need to be different than for the charge conjugated processes)
  - deviation from thermal equilibrium (otherwise asymmetry washed out)
- Different Baryogenesis theories
- Leptogenesis stems from a different problem in particle physics: The smallness of neutrino masses.



# The unbearable lightness of neutrino masses

- Compared to all other fermions, neutrinos are extremely light  $\Rightarrow$  Yukawa couplings of  $\sim 10^{-13}$  necessary for Dirac masses (cf.  $Y_e \sim 10^{-6}$ )
- Possible solution: Add three right-handed neutrinos  $N$  to the SM with Majorana mass terms at the GUT scale ( $\sim 10^{16}$  GeV), assume Yukawa couplings  $h$  similar to the other fermions

$$\delta\mathcal{L} = \bar{N}_i i \partial_\mu \gamma^\mu N_i - h_{i\alpha} \bar{N}_i \phi^\dagger \ell_\alpha - \frac{1}{2} M_i \bar{N}_i N_i^c + h.c.$$

- Diagonalizing the mass matrix leads to six mass eigenstates: three heavy ones ( $\sim M_i$ ) and three light ones ( $m_{ij}^\nu = -v^2 (h^T M^{-1} h)_{ij}$ )  $\Rightarrow$  See-saw mechanism
- The heavy  $N$ s violate lepton number, but did we not want a baryon asymmetry?



# Sphalerons: A black box

[’t Hooft ’76], [Rubakov, Shaposhnikov ’85]

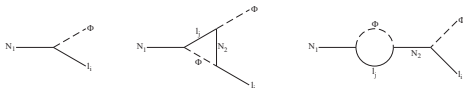
- Baryon (B) and lepton (L) numbers are not conserved in the SM due to a U(1) triangle anomaly
- At high  $T$  ( $\sim T_{EW} \sim 100$  GeV), thermal transitions between different vacua with different B and L numbers are possible: Sphalerons
- $\Delta B = \Delta L = 3$  for sphaleron processes, but  $B - L$  conserved  $\Rightarrow$  We need to violate  $B - L$ , otherwise sphaleron processes will wash out any existing asymmetry



# N decays

The lightest (heavy) Majorana neutrino  $N_1$  is an ideal candidate for baryogenesis:

- It decays out of equilibrium (no SM gauge interactions)
- It violates  $L$  and  $B - L$  ( $N \rightarrow \ell\phi^\dagger, \bar{\ell}\phi$ )
- Sphaleron processes convert lepton asymmetry partially to baryon asymmetry
- The generated baryon asymmetry is proportional to the CP asymmetry in  $N_1$ -decays: interference between tree level and one-loop diagrams:



- rough estimate for  $\epsilon_1$  in terms of neutrino masses:

$$\epsilon_1 \simeq -\frac{3}{16\pi} \frac{M_1}{(hh^\dagger)_{11} v^2} \text{Im}(h^* m_\nu h^\dagger)_{11} \simeq -\frac{3}{16\pi} \frac{M_1 m_3}{v^2} \sim 0.1 \frac{M_1}{M_3}$$

- For hierarchies like the quark sector ( $\frac{M_1}{M_3} \sim 10^{-5}$ )  $\Rightarrow \epsilon_1 \sim 10^{-6}$



# Baryon asymmetry

- Baryon asymmetry:  $\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = -d \epsilon_1 \kappa \sim 10^{-9}$ ,  
 with dilution factor  $d \sim 0.01$  (increase of photon number density),  
 efficiency factor  $\kappa$  typically  $\sim 0.1$  (Boltzmann equations);  
 baryogenesis temperature  $T_B \sim M_1 \sim 10^{10}$  GeV.
- Two parameters govern LG (neglecting flavor effects):  
 $M_1$  and 'effective'  $\nu$  mass  $\tilde{m}_1 = \frac{(hh^\dagger)_{11} v^2}{M_1}$
- $M_1$  decays in (out of) equilibrium if  $\Gamma_1 > H$  ( $\Gamma_1 < H$  with  $H$  Hubble parameter)  $\Rightarrow \tilde{m}_1 > m_* \simeq 10^{-3}$  eV ( $\tilde{m}_1 < m_* \simeq 10^{-3}$  eV)
- Numerical evaluation of Boltzmann equations shows: LG also possible close to equilibrium ( $\tilde{m}_1 > m_*$ )
- lower bound on reheating temperature  $T_R > 2 \times 10^9$  GeV



# Thermal field theory

- At zero temperature vacuum expectation values of operators:

$$\langle A \rangle = \langle 0|A|0 \rangle$$

- Two-point Green's function:

$$i\Delta(x-y) = \langle 0|T\{\phi(x)\phi(y)\}|0 \rangle$$

- At finite temperature ensemble weighted expectation values:

$$\langle A \rangle_\beta = \text{Tr}(\rho A) = \frac{1}{\text{Tr}(e^{-\beta H})} \sum_n \langle n|A|n \rangle e^{-\beta E_n}$$

- Two-point function:

$$i\Delta^{T>0}(x-y) = \frac{1}{\text{Tr}(e^{-\beta H})} \sum_n \langle n|T\{\phi_x\phi_y\}|n \rangle e^{-\beta E_n}$$



# Thermal propagators

- Using bare thermal propagators can give IR singularities and gauge dependent results
- cure: Hard Thermal Loop (HTL) resummation technique
- For soft momenta  $K \ll T$ , resummed propagators have to be used:

$$\begin{aligned}
 \text{---} \bullet \text{---} &= \text{---} + \text{---} \textcircled{\Sigma} \text{---} + \text{---} \textcircled{\Sigma} \textcircled{\Sigma} \text{---} + \dots \\
 i\Delta^* &= i\Delta + i\Delta(-i\Sigma)i\Delta + \dots = \frac{i}{\Delta^{-1} - \Sigma} = \frac{i}{K^2 - m^2 - \Sigma} \\
 \Rightarrow \text{Thermal masses } m_{\text{th}}^2(T) &:= m^2 + \Sigma
 \end{aligned}$$



# Thermal corrections to leptogenesis

Thermal corrections have been investigated [Giudice, Notari, Raidal, Riotto, Strumia '03]

- Renormalization of couplings at  $\sim 2\pi T$ , most importantly the top Yukawa and neutrino masses
- thermal corrections to decay and scattering processes, using HTL resummed propagators and thermal masses in the kinematics of the final states:
  - decays  $N \rightarrow HL$
  - $\Delta L = 2$  scatterings  $LH \rightarrow \overline{LH}$  and  $LL \rightarrow \overline{HH}$ , mediated by  $N_1$ . The  $N_1$  on-shell contributions are taken into account by decays and inverse decays and have to be subtracted
  - $\Delta L = 1$  scatterings involving the top quark and gauge bosons
- thermal corrections to the CP asymmetry  $\epsilon_{N_1}$



# Thermal masses by hand

- To understand thermal masses, consider the decay rate  $N \rightarrow LH$ .
- put in thermal masses “by hand”:

$$\gamma_D^{\text{eq}} = \int d\tilde{p}_N d\tilde{p}_L d\tilde{p}_H (2\pi)^4 \delta^4(P_N - P_L - P_H) |\mathcal{M}|^2 f_N (1 + f_H) (1 - f_L),$$

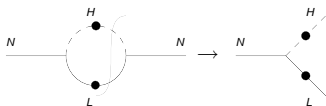
where  $d\tilde{p}_i = \frac{p_i^3}{(2\pi i)^3 2E_i}$

- Assume either
  - Fermi- and Bose-statistics
  - or Maxwell-Boltzmann statistics  $\Rightarrow$  enhancement and blocking factors  $(1 + f_H)(1 - f_L)$  become 1. Deviation typically  $\mathcal{O}(10\%)$ .



# May you put in thermal masses by hand?

- What is the consistent TFT-treatment of thermal masses?
- Calculate the  $N$  self energy at finite  $T$  and use TFT cutting rules [Weldon '83, Kobes and Semenoff '86]:



$$\Sigma = -g^2 T \sum_{p_L^0=2\pi i n T} \int \frac{d^3 p_L}{(2\pi)^3} S^*(P_L) D^*(P_H)$$



# approximate HTL-propagators

- $D^*(P_H) = \frac{1}{P_H^2 - m_H^2(T)}$  HTL-resummed Higgs propagator,
 
$$\frac{m_H^2(T)}{T^2} = \frac{3}{16}g_2^2 + \frac{1}{16}g_Y^2 + \frac{1}{4}y_t^2 + \frac{1}{2}\lambda,$$



- For demonstration purposes, do not take full HTL fermion propagator, but approximation  $S^*(P_L) = \frac{P_L}{P_L^2 - m_L^2(T)}$  (see next chapter),

$$\frac{m_L(T)^2}{T^2} = \frac{3}{32}g_2^2 + \frac{1}{32}g_Y^2 \text{ is given by gauge interactions}$$



# Interpretation of the discontinuity

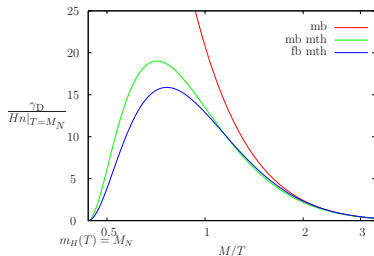
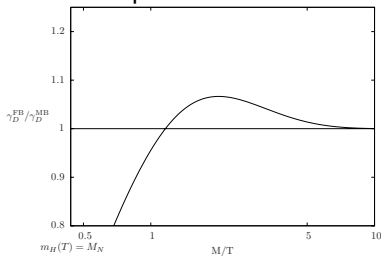
$$\begin{aligned} \text{tr}(\mathcal{P}_N \text{Im}\Sigma) &= - \int d\tilde{p}_L d\tilde{p}_H (2\pi)^4 \delta^4(P_N - P_H(T) - P_L(T)) \\ &\quad \times |\mathcal{M}|^2 (1 - f_L + f_H) \end{aligned}$$

- $1 - f_L + f_H = (1 - f_L)(1 + f_H) + f_L f_H$  includes both  $N \rightarrow LH$  and  $LH \rightarrow N$
- $\Gamma = -\frac{1}{2p_N^0} \text{tr}(\mathcal{P}_N \text{Im}\Sigma)$ ,  $\Gamma_D = (1 - f_N(p_N^0))\Gamma$ ,  $\Gamma_{ID} = f_N(p_N^0)\Gamma$
- result equals “per hand” treatment, propagators same structure as at  $T = 0$ , but
  - Full HTL Lepton propagator has a different structure  $\rightarrow$  quasi-particle structure
  - Fermi- and Bose distribution functions always appear in TFT calculations, even without thermal masses  $\Rightarrow$  Maxwell-Boltzmann appears not consistent



# decay rate $\gamma_D$

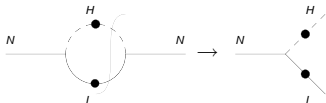
## Comparison of Fermi-Bose and Maxwell-Boltzmann



- $M_1 = 10^{10}$  GeV ,  $\tilde{m}_1 = 0.06$  eV ( $\Delta m_{atm}^2$ )
- Cutoff at  $M_N = m_H(T)$ , deviation of up to 20 %.



# Decay rate $\gamma_D$ : Including $m_L(T)$



- Due to its fermionic structure, the lepton propagator cannot be resummed like a scalar propagator, but yields a more complicated result.
- In the helicity eigenstate representation, it reads:

$$S^*(K) = \frac{1}{2D_+(K)}(\gamma_0 - \hat{\mathbf{k}} \cdot \boldsymbol{\gamma}) + \frac{1}{2D_-(K)}(\gamma_0 + \hat{\mathbf{k}} \cdot \boldsymbol{\gamma}),$$

where

$$D_{\pm}(K) = -k_0 \pm k + \frac{m_L(T)^2}{k} \left( \pm 1 - \frac{\pm k_0 - k}{2k} \ln \frac{k_0 + k}{k_0 - k} \right)$$

- The thermal lepton mass is given by gauge interactions:  $\frac{m_L(T)^2}{T^2} = \frac{3}{32}g_2^2 + \frac{1}{32}g_Y^2$

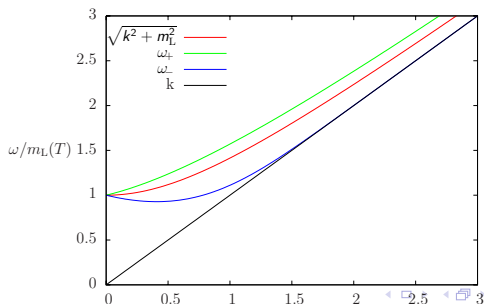


# Fermion dispersion relations

The propagator has two poles at  $D_{\pm}(\omega_{\pm}) = 0$  corresponding to two dispersion relations

$$\omega_{\pm} = \pm k \frac{W_{-1,0}(-\exp(-2\frac{k^2}{m_L^2} - 1)) - 1}{W_{-1,0}(-\exp(-2\frac{k^2}{m_L^2} - 1)) + 1} \quad (1)$$

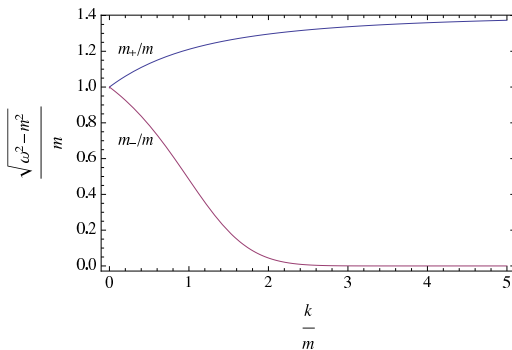
where  $W(s) = x$  is the Lambert W function, i.e. the inverse of  $s = xe^x$ , and  $W_0 \geq -1$  and  $W_{-1} \leq -1$  are its two real branches. The  $\omega_-$ -branch is the so-called plasmino.





# Quasiparticle masses revisited

- For  $k \rightarrow \infty$ :  $\omega_+(k) \rightarrow \sqrt{k^2 + 2m_L^2}$      $\omega_-(k) \rightarrow k$
- Using  $\omega_-$  and  $\omega_+$  corresponds to different thermal masses (ranging from 0 to  $\sqrt{2} m_L$ )  $\Rightarrow$  correction, threshold  $M_H = m_H + m_L$  relaxed.





# Decay rate for two lepton modes

- Calculating  $\Gamma_{\pm}$  using the optical theorem, one gets as matrix element

$$|\mathcal{M}_{\pm}|^2 = g^2 \frac{\omega_{\pm}^2 - k^2}{2m_L(T)^2} (\omega_{\pm} p_0 \mp \omega_{\pm} \eta),$$

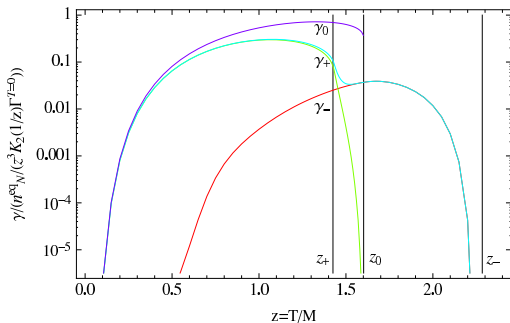
where  $\eta = \mathbf{k} \cdot \mathbf{p}/kp$  is the angle between neutrino and lepton.

- For comparison the one-mode approximation

$$|\mathcal{M}|^2 = g^2 K_{\mu} P^{\mu} = \frac{1}{2} (M_N^2 - m_L(T)^2 + m_H(T)^2)$$



# Comparing with one mode approximation



- decay density for the  $\pm$  modes and the one-mode approximation,  $M_N = 10^{10}\text{GeV}$ ,  $\tilde{m}_1 = 0.06\text{eV}$ .
- Thresholds are at  $M_N = m_H + \sqrt{2}m_L$ ,  $M_N = m_H$  and  $M_N = m_H + m_L$ .
- The deviation reaches one order of magnitude in the interesting temperature regime  $T \sim M$ .



# What needs to be done for a self-consistent implementation in the dynamics?

- When  $M_N \leq m_H(T)$ , the decay  $H \rightarrow NL$  opens up  $\Rightarrow$  Calculate  $\gamma_{H\pm}$ .
- Calculate the effect on the CP-asymmetries  $\epsilon_N(T)$ ,  $\epsilon_H(T)$ .
- Minimal self-consistent treatment without scatterings and gauge interactions, gives an idea of the effect.



# Conclusions

## • Summary:

- Putting in thermal masses “by hand” is a justifiable approximation for the  $N$  decay density  $\gamma_D$  to some accuracy.
- In the TFT treatment, Fermi-Bose distribution functions appear at the same order as thermal masses  $\rightarrow$  Maxwell-Boltzmann not consistent.
- Two lepton modes give corrections of one order of magnitude in the interesting temperature regime  $T \sim M_N$ .

## • Future work:

- Determine the dynamics of the minimally self-consistent scenario, only decays and inverse decays.
- Examine thermal corrections to other relevant processes, like  $LH$ -scatterings mediated by  $N$ ,  $NL$  scatterings involving the top quark or gauge bosons.
- Include thermal widths of quasiparticles.

Thank you for your attention!