Quasiparticles in Leptogenesis

Clemens Kießig

Max-Planck-Institut für Physik München
(Werner-Heisenberg-Institut)

based on work with Michael Plümacher

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Outline

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   - May you put in thermal masses “by hand”?
   - Fermionic quasiparticles
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Matter-Antimatter Asymmetry

- The universe today consists of matter and of practically no antimatter. Naively expected relic baryon density (annihilation into pions): $\frac{n_B}{n_\gamma} = \frac{n_{\bar{B}}}{n_\gamma} \sim 10^{-20}$
- Observed baryon asymmetry: $\eta_{\mathrm{B}} = \frac{n_{\mathrm{B}} - n_{\bar{\mathrm{B}}}}{n_\gamma} \sim 10^{-9}$
  $\Rightarrow$ Huge number! Explanation?
- A matter-antimatter asymmetry can be dynamically generated if Sakharov's conditions [1967] are fulfilled:
  - baryon number violation
  - C and CP violation (reaction rates need to be different than for the charge conjugated processes)
  - deviation from thermal equilibrium (otherwise asymmetry washed out)
- Different Baryogenesis theories
- Leptogenesis stems from a different problem in particle physics: The smallness of neutrino masses.

The unbearable lightness of neutrino masses

- Compared to all other fermions, neutrinos are extremely light $\Rightarrow$ Yukawa couplings of $\sim 10^{-13}$ necessary for Dirac masses (cf. $Y_e \sim 10^{-6}$)
- Possible solution: Add three right-handed neutrinos N to the SM with Majorana mass terms at the GUT scale ($\sim 10^{16}$ GeV), assume Yukawa couplings h similar to the other fermions

$$\delta\mathcal{L} = \bar{N}_i i\partial_\mu \gamma^\mu N_i - h_{i\alpha}\bar{N}_i \phi^\dagger \ell_\alpha - \frac{1}{2} M_i \bar{N}_i N_i^c + h.c.$$

- Diagonalizing the mass matrix leads to six mass eigenstates: three heavy ones ($\sim M_i$) and three light ones ($m_{ij}^\nu = -v^2 (h^T M^{-1} h)_{ij}$) $\Rightarrow$ See-saw mechanism
- The heavy Ns violate lepton number, but did we not want a baryon asymmetry?

Sphalerons: A black box

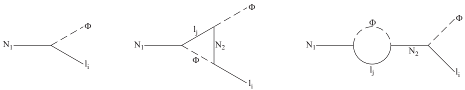
['t Hooft '76], [Rubakov, Shaposhnikov '85]

- Baryon (B) and lepton (L) numbers are not conserved in the SM due to a U(1) triangle anomaly
- At high T ($\sim T_{EW} \sim 100$ GeV), thermal transitions between different vacua with different B and L numbers are possible: Sphalerons
- $\Delta B = \Delta L = 3$ for sphaleron processes, but $B - L$ conserved $\Rightarrow$ We need to violate $B - L$, otherwise sphaleron processes will wash out any existing asymmetry

N decays

The lightest (heavy) Majorana neutrino N_1 is an ideal candidate for baryogenesis:

- It decays out of equilibrium (no SM gauge interactions)
- It violates L and $B-L$ ($N \rightarrow \ell\phi^\dagger, \overline{\ell}\phi$)
- Sphaleron processes convert lepton asymmetry partially to baryon asymmetry
- The generated baryon asymmetry is proportional to the CP asymmetry in N_1-decays: interference between tree level and one-loop diagrams:



- rough estimate for ϵ_1 in terms of neutrino masses:

$$\epsilon_1 \simeq -\frac{3}{16\pi}\frac{M_1}{(hh^\dagger)_{11}v^2}\mathrm{Im}(h^* m_\nu h^\dagger)_{11} \simeq -\frac{3}{16\pi}\frac{M_1 m_3}{v^2} \sim 0.1\frac{M_1}{M_3}$$

- For hierarchies like the quark sector ($\frac{M_1}{M_3} \sim 10^{-5}$) $\Rightarrow \epsilon_1 \sim 10^{-6}$

Baryon asymmetry

- Baryon asymmetry: $\eta_B = \frac{n_{\mathrm{B}} - n_{\bar{\mathrm{B}}}}{n_\gamma} = -d\ \epsilon_1\ \kappa \sim 10^{-9}$,
  with dilution factor $d \sim 0.01$ (increase of photon number density),
  efficiency factor κ typically ~ 0.1 (Boltzmann equations);
  baryogenesis temperature $T_B \sim M_1 \sim 10^{10}$ GeV.
- Two parameters govern LG (neglecting flavor effects):
  M_1 and 'effective' ν mass $\tilde{m}_1 = \frac{(hh^\dagger)_{11} v^2}{M_1}$
- N_1 decays in (out of) equilibrium if $\Gamma_1 > H$ ($\Gamma_1 < H$ with H Hubble parameter) $\Rightarrow \tilde{m}_1 > m_* \simeq 10^{-3}$ eV ($\tilde{m}_1 < m_* \simeq 10^{-3}$ eV)
- Numerical evaluation of Boltzmann equations shows: LG also possible close to equilibrium ($\tilde{m}_1 > m_*$)
- lower bound on reheating temperature $T_R > 2 \times 10^9$ GeV

Thermal field theory

- At zero temperature vacuum expectation values of operators:

$$\langle A\rangle = \langle 0|A|0\rangle$$

- Two-point Green's function:

$$i\Delta(x-y) = \langle 0|T\{\phi(x)\phi(y)\}|0\rangle$$

- At finite temperature ensemble weighted expectation values:

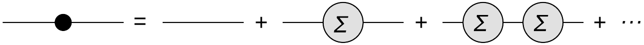
$$\langle A\rangle_\beta = Tr(\rho A) = \frac{1}{Tr(e^{-\beta H})}\Sigma_n \langle n|A|n\rangle\, e^{-\beta E_n}$$

- Two-point function:

$$i\Delta^{T>0}(x-y) = \frac{1}{Tr(e^{-\beta H})}\Sigma_n \langle n|T\{\phi_x\phi_y\}|n\rangle\, e^{-\beta E_n}$$

Thermal propagators

- Using bare thermal propagators can give IR singularities and gauge dependent results
- cure: Hard Thermal Loop (HTL) resummation technique
- For soft momenta $K \ll T$, resummed propagators have to be used:



$$i\Delta^* = i\Delta + i\Delta(-i\Sigma)i\Delta + \ldots = \frac{i}{\Delta^{-1} - \Sigma} = \frac{i}{K^2 - m^2 - \Sigma}$$

$\Rightarrow$ Thermal masses $m_{\text{th}}^2(T) := m^2 + \Sigma$

Thermal corrections to leptogenesis

Thermal corrections have been investigated [Giudice, Notari, Raidal, Riotto, Strumia '03]

- Renormalization of couplings at $\sim 2\pi T$, most importantly the top Yukawa and neutrino masses
- thermal corrections to decay and scattering processes, using HTL resummed propagators and thermal masses in the kinematics of the final states:
  - decays $N \to HL$
  - $\Delta L = 2$ scatterings $LH \to \overline{LH}$ and $LL \to \overline{HH}$, mediated by N_1. The N_1 on-shell contributions are taken into account by decays and inverse decays and have to be subtracted
  - $\Delta L = 1$ scatterings involving the top quark and gauge bosons
- thermal corrections to the CP asymmetry ϵ_{N_1}

Thermal masses by hand

- To understand thermal masses, consider the decay rate $N \to LH$.
- put in thermal masses "by hand":

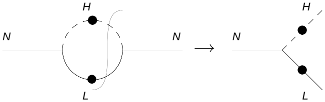
$$\gamma_D^{eq} = \int \mathrm{d}\tilde{p}_N \mathrm{d}\tilde{p}_L \mathrm{d}\tilde{p}_H (2\pi)^4 \delta^4(P_N - P_L - P_H) |\mathcal{M}|^2 f_N (1+f_H)(1-f_L),$$

where $\mathrm{d}\tilde{p}_i = \frac{p_i^3}{(2pi)^3 2E_i}$

- Assume either
  - Fermi- and Bose-statistics
  - or Maxwell-Boltzmann statistics $\Rightarrow$ enhancement and blocking factors $(1+f_H)(1-f_L)$ become 1. Deviation typically $\mathcal{O}(10\%)$.

May you put in thermal masses by hand?

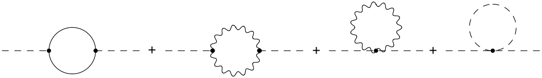
- What is the consistent TFT-treatment of thermal masses?
- Calculate the N self energy at finite T and use TFT cutting rules [Weldon '83, Kobes and Semenoff '86]:



$$\Sigma = -g^2 T \sum_{p_L^0 = 2\pi i n T} \int \frac{\mathrm{d}^3 p_L}{(2\pi)^3} S^*(P_L) D^*(P_H)$$

approximate HTL-propagators

- $D^*(P_H) = \frac{1}{P_H^2 - m_H^2(T)}$ HTL-resummed Higgs propagator,
  $\frac{m_H^2(T)}{T^2} = \frac{3}{16} g_2^2 + \frac{1}{16} g_Y^2 + \frac{1}{4} y_t^2 + \frac{1}{2} \lambda,$



- For demonstration purposes, do not take full HTL fermion propagator, but approximation $S^*(P_L) = \frac{\not{P}_L}{P_L^2 - m_L^2(T)}$ (see next chapter),
  $\frac{m_L(T)^2}{T^2} = \frac{3}{32} g_2^2 + \frac{1}{32} g_Y^2$ is given by gauge interations

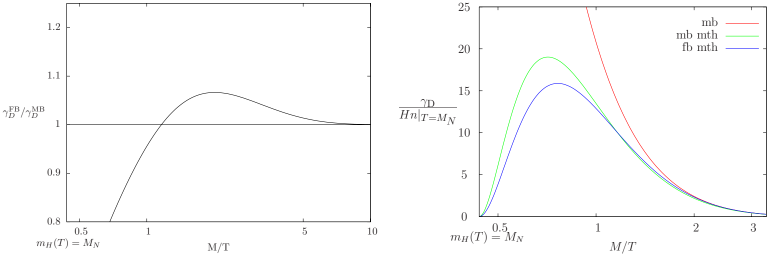
Interpretation of the discontinuity

$$\mathrm{tr}(\not{P}_N \mathrm{Im}\Sigma) = -\int \mathrm{d}\tilde{p}_L \mathrm{d}\tilde{p}_H (2\pi)^4 \delta^4(P_N - P_H(T) - P_L(T)) \times |\mathcal{M}|^2 (1 - f_L + f_H)$$

- $1 - f_L + f_H = (1 - f_L)(1 + f_H) + f_L f_H$ includes both $N \rightarrow LH$ and $LH \rightarrow N$
- $\Gamma = -\frac{1}{2p_N^0} tr(\not{P}_N Im\Sigma)$, $\Gamma_D = (1 - f_N(p_N^0))\Gamma$, $\Gamma_{ID} = f_N(p_N^0)\Gamma$
- result equals "per hand" treatment, propagators same structure as at $T = 0$, but
  - Full HTL Lepton propagator has a different structure $\rightarrow$ quasi-particle structure
  - Fermi- and Bose distribution functions always appear in TFT calculations, even without thermal masses $\Rightarrow$ Maxwell-Boltzmann appears not consistent

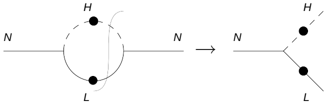
decay rate γ_D

Comparison of Fermi-Bose and Maxwell-Boltzmann



- $M_1 = 10^{10}$ GeV , $\tilde{m}_1 = 0.06$ eV (Δm^2_{atm})
- Cutoff at $M_N = m_H(T)$, deviation of up to 20 %.

Decay rate γ_D: Including $m_L(T)$



- Due to its fermionic structure, the lepton propagator cannot be resummed like a scalar propagator, but yields a more complicated result.
- In the helicity eigenstate representation, it reads:

$$S^*(K) = \frac{1}{2D_+(K)}(\gamma_0 - \hat{\mathbf{k}}\cdot\gamma) + \frac{1}{2D_-(K)}(\gamma_0 + \hat{\mathbf{k}}\cdot\gamma),$$

where

$$D_\pm(K) = -k_0 \pm k + \frac{m_L(T)^2}{k}\left(\pm 1 - \frac{\pm k_0 - k}{2k}\ln\frac{k_0+k}{k_0-k}\right)$$

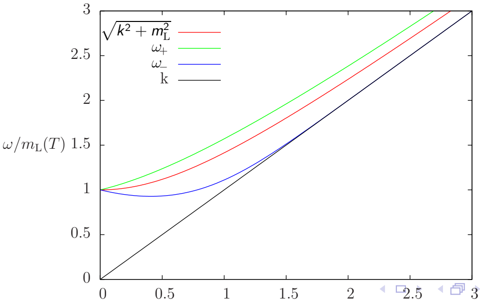
- The thermal lepton mass is given by gauge interactions: $\frac{m_L(T)^2}{T^2} = \frac{3}{32}g_2^2 + \frac{1}{32}g_Y^2$

Fermion dispersion relations

The propagator has two poles at $D_\pm(\omega_\pm) = 0$ corresponding to two dispersion relations

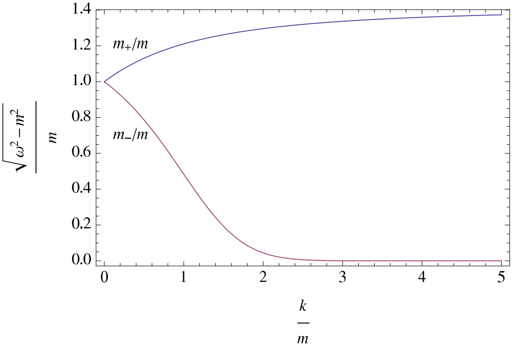
$$\omega_\pm = \pm k \frac{W_{-1,0}(-\exp(-2\frac{k^2}{m_L^2} - 1)) - 1}{W_{-1,0}(-\exp(-2\frac{k^2}{m_L^2} - 1)) + 1} \qquad (1)$$

where $W(s) = x$ is the Lambert W function, i.e. the inverse of $s = x\mathrm{e}^x$, and $W_0 \geq -1$ and $W_{-1} \leq -1$ are its two real branches. The ω_--branch is the so-called plasmino.



Quasiparticle masses revisited

- For $k \to \infty$: $\omega_+(k) \to \sqrt{k^2 + 2m_L^2}$ $\omega_-(k) \to k$
- Using ω_- and ω_+ corresponds to different thermal masses (ranging from 0 to $\sqrt{2}\ m_L$) $\Rightarrow$ correction, threshold $M_H = m_H + m_L$ relaxed.



Decay rate for two lepton modes

- Calculating $\Gamma_\pm$ using the optical theorem, one gets as matrix element

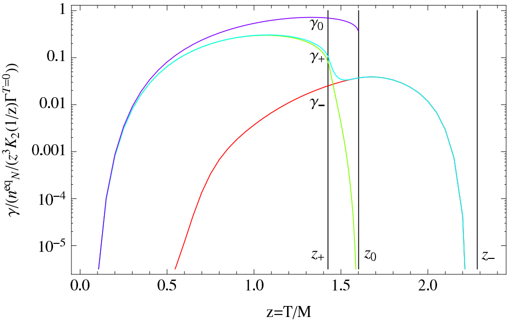
$$|\mathcal{M}_\pm|^2 = g^2 \frac{\omega_\pm^2 - k^2}{2 m_L(T)^2} \left(\omega_\pm p_0 \mp \omega_\pm \eta\right),$$

where $\eta = \mathbf{k} \cdot \mathbf{p} / kp$ is the angle between neutrino and lepton.

- For comparison the one-mode approximation

$$|\mathcal{M}|^2 = g^2 K_\mu P^\mu = \frac{1}{2}\left(M_N^2 - m_L(T)^2 + m_H(T)^2\right)$$

Comparing with one mode approximation



- decay density for the $\pm$ modes and the one-mode approximation, $M_N = 10^{10}\mathrm{GeV}$, $\tilde{m}_1 = 0.06\mathrm{eV}$.
- Thresholds are at $M_N = m_H + \sqrt{2} m_L$, $M_N = m_H$ and $M_N = m_H + m_L$.
- The deviation reaches one order of magnitude in the interesting temperature regime $T \sim M$.

What needs to be done for a self-consistent implementation in the dynamics?

- When $M_N \leq m_H(T)$, the decay $H \rightarrow NL$ opens up $\Rightarrow$ Calculate $\gamma_{H\pm}$.
- Calculate the effect on the CP-asymmetries $\epsilon_N(T)$, $\epsilon_H(T)$.
- Minimal self-consistent treatment without scatterings and gauge interactions, gives an idea of the effect.

Conclusions

- Summary:
  - Putting in thermal masses "by hand" is a justifiable approximation for the N decay density γ_{D} to some accuracy.
  - In the TFT treatment, Fermi-Bose distribution functions appear at the same order as thermal masses $\longrightarrow$ Maxwell-Boltzmann not consistent.
  - Two lepton modes give corrections of one order of magnitude in the interesting temperature regime $T \sim M_N$.
- Future work:
  - Determine the dynamics of the minimally self-consistent scenario, only decays and inverse decays.
  - Examine thermal corrections to other relevant processes, like LH-scatterings mediated by N, NL scatterings involving the top quark or gauge bosons.
  - Include thermal widths of quasiparticles.

Thank you for your attention!