An adjustable cosmological constant

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1. Introduction



The cosmological constant was introduced almost 93 years ago by Einstein.

Since the spontaneous symmetry breaking is known, Veltman [PRL 34 (1977) 777] commented that the vacuum energy arising in spontaneous symmetry breaking adds to the cosmological constant, basically raising a question on the naturalness of setting it to zero.

Even before considering the tree level c.c., the loop correction to the vacuum energy was a problem since the early days of quantum mechanics.

Here, we do not rely on some anthropic solution [Weinberg, PRL 59 (1988) 2607; V. Agrawal, S. M. Barr, J. Donoghue, PRD 57 (1998) 5480].



So, we must consider the cosmological constant generically, at tree level, and also at loop levels unless the following type of diagrams are forbidden:



The c.c. is a serious fine-tuning problem. In 4D, we do not find Any symmetry such that the c.c. is forbidden.



If a symmetry is present in changing Λ , one may try a scalar potential:



So, a solution is not easily realizable in 4D. The c.c. problem must also take into account the onset of spontaneous symmetry breaking.



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Mass scales of physics:

Planck scale: 2.44×10^{18} GeVGUT scale: 2×10^{16} GeVInflation scale: 10^{16} GeV, down to EWAxion scale: $\approx 10^{12} - 10^{11}$ GeVHidden scale: 10^{13} GeVEW scale: ≈ 100 GeVQCD scale:1 GeVNuclear physics scale:10 MeVelectron mass scale:0.5 MeVaccelerating universe: $(0.002 \text{ eV})^4$

Even though we have a tree level c.c. solution at an EW scale, still 10⁻⁶⁰ fine-tuning is needed.



2. Probability amplitude



When we consider QM, we talk in terms of the probability amplitude: The initial state |i> to transform to a final state |f>. So, E. Baum[PLB 133 (1983) 185] and S. Hawking[PLB 134 (1984) 403] considered the Euclidian action with the

R and Λ terms only.

The Euclidian action integral has the form

$$\exp(-\widetilde{I}) = \exp\left(\frac{3\pi M_P^2}{\Lambda}\right)$$

So, $\Lambda = 0^+$ is has a very large value, the probability for it is close to 1. But, there are questions regarding to this solution.



S. Hawking [PLB 134 (1984) 403] states

"My proposal requires that a variable effective c.c. be genarated in some manner. One possibility would be to include the values of the c.c. in the variables that are integrated over in the path integral."

Explicitly, a scalar field without the kinetic energy term is considered: $A_{\mu\nu\rho}$ (F_{µνρσ} field strength),





or

It is the action integral. Another question is

"How do we assign the initial state?""How does the needed primary inflation come about in this scenario?""How does it fit to the current dark energy?"

So the c.c. solution needs to explain other two c.c. related questions also.

The idea of no KE term



cannot explain all the above questions. We must introduce the KE term



Then, we consider the following type



In this case, $H_{\mu\nu\rho\sigma}$ field is constant in 4D, and is not of use any more. If we want to use $H_{\mu\nu\rho\sigma}$ field, we must work beyond 4D.



3. 5D Self-tuning and Initial state



In 2000, self-tuning solutions have been tried in RS type models. Here, I just mention the initial try and its failure. In RS-I model,

Arkani-Hamed, Dimopoulos, Kaloper, Sundrum [hep-th/0001197] Kachru, Schulz, Silverstein [hep-th/0001206]

Bulk action is fixed with specific magnitude of coupling. SM fields live at the brane. The loop corrections at the brane does not change the solution. A singularity is present at y_s . In a sense, the vacuum energy at the SM brane should be cured.

The singularity is cured by inserting a brane there. Then, to cancel the contribution of the SM brane, one must fine-tune the contribution from the singularity. A fine-tuning again. Forste, Lalak, Lavignac, Nilles [hep-th/0002264]

Furthermore, the no-go theorem exists under some plausible conditions (usual KE term, existence of 4D gravity): Csaki, Erlich, Grojean, Hollowood [hep-th/0004133]



We will distinguish

 Λ s :

Barred ones and the rest.

$\overline{\Lambda}$	= one	obtained	from	8 _{µv}
Λ	= one	obtained	from	$T_{\mu\nu}$





ADKS, KSS considered

In addition, we want to go beyond. I.e. we do not specify the bulk action. But, we allow non-standard kinetic energy term.





The Kim-Kyae-Lee solution [hep-th/0001118; hep-th/0101027] with a RS-II type solution with

Η_{μνρσ}

$$-I_E = \int d^5 x_E \sqrt{g_{(5)}} \left(\frac{1}{2}R_{(5)} - \frac{2\cdot 4!}{H^2} - \Lambda_b - \Lambda_1 \delta(y)\right)$$

= $\int dy \int d^4 x_E \left\{ -\Psi^4 \Lambda_1 \delta(y) + \frac{1}{2}R\Psi^2 + 4\Psi^3 \Psi'' + 6\Psi^2 (\Psi')^2 + \frac{2\cdot 4!\Psi^4}{H^2} - \Psi^4 \Lambda_b \right\}$

We take the following metric ansatz,

$$ds^{2} = \beta^{2}(y)\eta_{\mu\nu}dx^{\mu}dx^{\nu} + dy^{2}$$
$$\eta_{\mu\nu} = diag.(-1,+1,+1,+1,+1)$$



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The KE term with H² is not developing a VEV in the low energy theory, i.e. in the long wavelength limit $(\partial_{\mu}A_{\nu\rho\sigma}) \rightarrow 0$. Fortunately, however, we allow bare c.c. in the bulk. So, even with <H²>=0, H² can be moved to the denominator,

$$\frac{1}{H^2}$$
, with $\langle H^2 \rangle \neq 0$

The field equation and the Bianchi identity are satisfied with

$$\partial_{M}\left(\sqrt{g}\frac{H^{MNPQ}}{H^{4}}\right) = 0, \qquad \varepsilon^{RMNPQ}\left[\sqrt{g_{(5)}}H_{MNPQ}\right] = 0$$





With this Lagrangian, there exists a self-tuning solution:

$$\beta(y) = \left(\frac{k}{a}\right)^{1/4} \frac{1}{\cosh^{1/4}(4k \mid y \mid +c)},$$
$$k = \sqrt{\frac{-\Lambda_b}{6}}, \qquad a = \sqrt{\frac{1}{6 \mid A \mid}}$$

But there are nearby dS and AdS solutions also.

How does one choose the flat one?

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Two Einstein equations we considered in the bulk are

$$(\mu\nu): \quad -3\frac{\overline{\Lambda}}{\beta^2} + 3\left(\frac{\beta'}{\beta}\right)^2 + 3\frac{\beta''}{\beta} = -\Lambda_b - 2 \cdot 4! \left(\frac{3}{H^2}\right)$$
$$(55): \quad -\frac{6\overline{\Lambda}}{\beta^2} + 6\left(\frac{\beta'}{\beta}\right)^2 = -\Lambda_b - 2 \cdot 4! \left(\frac{1}{H^2}\right)$$



It is easy to show the existence of the nearby dS and AdS solutions:

Around here, we can try

$$Y = A\left[\sec h(ky + c_0) + c(y)\right]$$

where

 $Y = \beta^4, \quad \Lambda_b = -m^2 k^2$ $-\frac{1}{4}Y'' = 3\overline{\Lambda}\sqrt{Y} + \frac{2m^2 k^2}{3}Y - \frac{8}{3h}Y^3 - \frac{\Lambda_1}{3}\delta(y)Y$ $\longrightarrow m^2 = \frac{3}{8}, \quad \frac{16A^2}{3h}k^2$





In obtaining the above FLAT solution, the following B.C. was needed



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4. Probability amplitude with self-tuning solution



Hawking calculated the probability amplitude from

$$\left\langle \overline{\Lambda}_{f} \mid I \right\rangle = \int d[g] e^{-I_{E}[g]}$$

and the volume integral is the largest for $\Lambda = 0^+$. It is not satisfactory in two accounts. Firstly, the initial state is not clearly given. Second, it is not clear how the primodial inflation is taken into account.

One crucial defect is that he considered only the c.c. term. However, there arises vacuum energy from particle physics Lagrangian also. This may change his view completely.

Here, I discuss the initial state and the probability amplitude in order.





In the RS-II model, matter fields and gauge fields live at the brane localized at y=0. Inflation takes place at the y=0 brane. We have seen that the flat solution is allowed for

$$-\sqrt{-6\Lambda_b} \le \Lambda_1 \le +\sqrt{-6\Lambda_b}$$
 Eq. (1)

The inflation or dS expansion is taking place in our scheme for the parameter range such that the flat solution is forbidden.

$$|\Lambda_1| \ge +\sqrt{-6\Lambda_b}$$

For this range of Λ_1 , inflation is going on. Λ_1 , however, can change, for example by the waterfall field in the hybrid inflation at the brane, so that Λ_1 , falls in the region of Eq. (1).





This situation is shown below, from the initial $\Lambda_1 : \Lambda_1 > (6\Lambda_b)^{1/2}$



The roll over to the (flat, dS, AdS) region is like choosing the initial condition |I>. It is an eigenstate of Hamiltonian. It is a kind of filtering in QM.

We start the universe from this state, after finishing inflation at dS.





The picture is the following.

Usually, the inflation is taking place at the brane by the slow roll along the inflaton direction: chaotic, hybrid, etc.



An example with natural inflation:



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Hawking's probability amplitude must take into account the particle physics action also. In our case, the bulk action. For $H_{\mu\nu\rho\sigma}$, the surface term must be considered. There has been discussions on this point by M. Duff [PLB226 (1989) 36] and Z. C. Wu [PLB659 (2008) 891]. Duff just included the surface term, and Wu took into account the topology of the solution. And Duff got the opposite sign from Hawking and Wu got the same sign.

$$\begin{split} -I_E &= \int d^5 x_E \sqrt{g_{(5)}} \left(\frac{1}{2}R_{(5)} - \frac{2 \cdot 4!}{H^2} - \Lambda_b - \Lambda_1 \delta(y)\right) \\ &= \int dy \int d^4 x_E \left\{ -\Psi^4 \Lambda_1 \delta(y) + \frac{1}{2} R \Psi^2 + 4 \Psi^3 \Psi'' \right. \\ &+ 6 \Psi^2 (\Psi')^2 + \frac{2 \cdot 4! \Psi^4}{H^2} - \Psi^4 \Lambda_b \right\} \\ &- I_{\text{Surface}} \supset \int dy \int d^4 x_E \left\{ \rho \frac{2 \Psi^8}{H^2} \epsilon^{\mu \nu \rho \sigma} H_{\mu \nu \rho \sigma} - \frac{\Psi^4}{2} \rho^2 \right\} \end{split}$$

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Duff found the opposite sign from that of Hawking and Wu agrees with Hawking. We will present both cases with 1/H².

Hawking's basic argument was the size of the Euclidian volume. The dS space volume is finite, the flat space volume is infinite, and the AdS space volume is even more infinite. If the AdS is not considered, the flat space wins in the size of the volume. If we consider only the $1/\Lambda$ term, the AdS wins in magnitude but the sign is opposite from the dS; thus the flat space is chosen.



If we consider the sizes of volumes, AdS wins over flat even though both are infinite. For the flat volume, we take the Λ =0 limit of the dS case. For the AdS volume, we need to regularize the infinity to compare different cases of Λ 's.



We will not pay much attention to the $1/\Lambda$ term.





Euclidian AdS Metric

The metric of Euclidean maximally-symmetric space: (k = -1, 0 and 1 for AdS, flat and dS, respectively)

$$ds^{2} = a^{2} \left(\frac{dr^{2}}{1 - kr^{2}} + r^{2} d\Omega_{n-1}^{2} \right)$$

Since the Weyl tensor is known to be zero in this space, it can be connected to the flat space with a Weyl transf.

$$ds^{2} = a^{2} \left(\frac{dr^{2}}{1 - kr^{2}} + r^{2} d\Omega_{n-1}^{2} \right) = a^{2} f^{2}(\eta) (d\eta^{2} + \eta^{2} d\Omega_{n-1}^{2})$$

Here, $f(\eta) = r/\eta$ and $dr/\sqrt{1-kr^2} = f(\eta)d\eta = (r/\eta)d\eta$. To obtain $f(\eta)$, we should integrate above expression. (From here, we only consider the case of k = -1, AdS.)

$$\ln \eta = \int \frac{dr}{r\sqrt{1-kr^2}} = -\int \frac{dy}{y\sqrt{1+1/y^2}} = -\operatorname{arcsinh}(1/r)$$

Thus, $f(\eta) = r/\eta = 2/(1 - \eta^2)$. We know that the Ricci scalar R' with metric $g'_{\mu\nu} = a^2 f^2(\eta) g_{\mu\nu}$ is given by

$$R' = a^{-2} f^{-2} \left(R - 2(n-1) \nabla^2 (\ln f) - (n-1)(n-2) \left(\frac{f'}{f}\right)^2 \right).$$

In our case, R = 0 because it's flat. From now, we will call R' as R. Use the convention for c.c.: $R_{\mu\nu} = \bar{\Lambda}g_{\mu\nu}$, $R = n\bar{\Lambda}$. Thus, $\bar{\Lambda} = -2(n-1)/a^2$ for n-dimensional Euclidean AdS.

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n-dimensional Euclidean AdS volume

For convenience, we will use the metric of AdS in the form

$$ds^{2} = a^{2}f^{2}(\eta)(d\eta^{2} + \eta^{2}d\Omega_{n-1}^{2}).$$

Then volume of AdS can be written as

$$\begin{split} V_{\text{AdS}^n} &= a^n \int d^n x \left(\frac{2}{1-\eta^2}\right)^n \\ &= a^n \int_0^1 d\eta \ \eta^{n-1} V_{S^{n-1}} 2^n (1-\eta^2)^{-n} \\ &= (2a)^n V_{S^{n-1}} \frac{1}{2} \int_0^1 d\xi \ \xi^{n/2-1} (1-\xi)^{-n} \\ &= (2a)^n V_{S^{n-1}} \frac{1}{2} B(n/2, 1-n) \\ &= \left(2a\right)^n \frac{2\pi^{n/2}}{\Gamma(n/2)} \frac{1}{2} \frac{\Gamma(n/2)\Gamma(1-n)}{\Gamma(1-n/2)} \\ &= \left(\frac{8(n-1)\pi}{|\overline{\Lambda}|}\right)^{n/2} \frac{\Gamma(1-n)}{\Gamma(1-n/2)}. \end{split}$$





Also, the 5th dimension integration gives a contribution.

$$-I_{E} \propto \begin{bmatrix} M_{P}^{4}a_{dS}^{4} \times O(ky_{H}), & dS \\ M_{P}^{4}a_{dS}^{4} \times O(\infty) \times O(ky_{H}), & Flat \\ M_{P}^{4}a_{dS}^{4} \times O(\infty) \times O(ky_{P}), & AdS \end{bmatrix}$$

We integrate out 4D and the 5th space y. Here, the brane tension Λ_1 contributes also. For the coefficient of Λ -bar⁻² to be positive, the following is required

$$\tanh(c_0) \sec h^2(c_0) \le \frac{k}{3} F(c_0/k, d_m)$$





With Duff's form,

$$F(c_0/k, d_m) \simeq \int_0^{d_m} dy \left(\frac{14}{8} \operatorname{sech} + \frac{22}{8} \operatorname{sech}^3\right)$$

= $+\frac{14}{8k} \sin^{-1}(\tanh) - \frac{14}{8k} \sin^{-1}(\tanh c_0)$
+ $\frac{22}{8} \frac{1}{2k} \sinh \operatorname{sech}^2 - \frac{22}{8} \frac{1}{2k} \sinh c_0 \operatorname{sech}^2 c_0$
+ $\frac{22}{8} \frac{1}{2k} \tan^{-1}(\sinh) - \frac{22}{8} \frac{1}{2k} \tan^{-1}(\sinh c_0)$

This integral turns out to be positive. For example, for the flat space, the c_0 independent part is $(9/2k)(\pi/2)$.

Even with Wu's sign, we have a positive one: $(3/8k)(\pi/2)$.

$$e^{-I_E} \propto \exp\left[rac{ ext{positive number}}{\overline{\Lambda}^2}
ight]$$
 ...A-bar=0 is preferred









Why do they obtain different results?

Because they did not consider the correct vacuum.

For two antisymmetric indices from { μ , ν , ρ , σ }, there are six (₄C₂) independent 2nd-rank antisymmetric gauge functions, and A transforms as

$$\mathsf{A}_{\mu\nu\rho} \longrightarrow \mathsf{A}_{\mu\nu\rho} - \partial_{\mu}\Lambda_{\nu\rho} - \partial_{\nu}\Lambda_{\rho\mu} - \partial_{\rho}\Lambda_{\mu\nu}$$

Choose three out six gauge functions.





We can do with 1/H² term, but let me show the idea with H² term since the line by line goes parallel. Thus, there exist maps of

 $S_3 \rightarrow S_3.$

Let $A_{\mu\nu\rho}$ be a pure gauge

$$\Lambda_{\mu\nu} = \frac{x_{\mu} \otimes p_{\nu}}{(r^2 + \rho^2)^2}$$
$$\partial_{\mu}\Lambda_{\nu\rho} \propto \frac{1}{r^3}, \text{ pure-gauge} \quad A_{\mu\nu\rho}$$

g(x) is a pure gauge form. $H_{\mu\nu\rho\sigma} \sim 1/r^4$. ρ =size of instanton





Pontryagin integers

$$\frac{1}{32\pi^2} \int d^4 x \ \varepsilon_{\mu\nu\rho\sigma} H_{\mu\nu\rho\sigma} = n = \text{int eger}$$
$$|\alpha\rangle = \sum_{n=-\infty}^{\infty} e^{in\alpha} |n\rangle$$

Thus, a gauge invariant $|\alpha>$ vacuum can be considered. In the $|\alpha>$ vacuum, we consider the following interaction in the Euclidian space,

$$i\frac{\alpha}{32\pi^2}\int d^4x \ \varepsilon_{\mu\nu\rho\sigma}H_{\mu\nu\rho\sigma}$$

 $0 \le \alpha \le \pi$: $\alpha = 0$ is the minimum [Vafa-Witten]. $\alpha = \pi$ is the maximum.





 α =0 is the one calculated by Wu. α = π is the one calculated by Duff.

Thus, the gauge invariant $|\alpha>$ vacuum choose any value between $(9/2k)(\pi/2)$ and $(3/8k)(\pi/2)$ for the c₀ independent part.

Namely, working in the $|\alpha>$ vacuum, we do not encounter any inconsistency, and for the region of parameters we chose, we have

$$e^{-I_E} \propto \exp\left[\frac{\text{positive number}}{\overline{\Lambda}^2}\right]$$





Саре Тоwn, **2 Feb**. 2010



In conclusion, we observed the following :

- The c.c. problem may be understandable at D>4.
- Three c.c. problems should be addressed.
- The initial state of the Universe should be given properly.
- A brane helps in solving the vanishing c.c. problem, since the loop effects of brane is not important to bulk physics.
- The action integral is dominated from the part. phys. part. and has the amplitude proportional to $\exp[\#/\Lambda$ -bar²]
- Near Λ -bar=0, AdS space is preferred. But slightly outside Λ -bar=0, dS space is preferred.
- The current acceleration should be addressed. The quintessential-axion idea may be useful.





Our specific example with the three index gauge field in the KKL model :

- The gauge invariant vacuum $|\alpha>$ is considered.
- Then the amount of the surface term α to insert is a parameter in the theory, like θ in QCD..
- For any value of α , there exists a finite range of parameters such that Λ -bar=0⁻ is chosen.
- If α is made dynamical as the axion in QCD, then the probability amplitude choosing Λ -bar=0⁻ is the value given by Wu.







