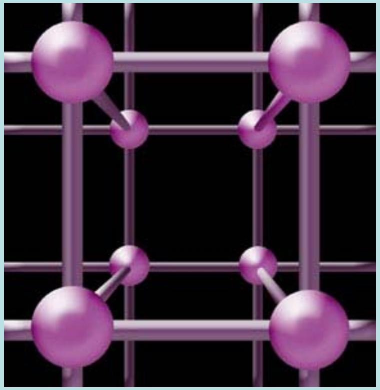
TeV Physics and Conformality

Cape Town February 1, 2010

1. Collaboration with George Fleming and Ethan Neil

1) arXiv:0712.0609, PRL 100, 171607 , 2008

2) arXiv:0901.3766 PR D79, 076010, 2009



2. LSD collaboration **J. C. Osborn, R. Babich, R. C. Brower, M. A. Clark, C. Rebbi, D. Schaich, M. Cheng, T. Luu, R. Soltz, P. M. Vranas, T. Appelquist, G. T. Fleming, E. T. Neil, Meifeng Lin**

arXiv:0910.2224 → PRL

New Strong Dynamics

Electroweak breaking

- Technicolor, Higgless theories, ADS/CFT approaches
- Near-conformal infrared behavior: walking technicolor

New, SM-singlet Sector

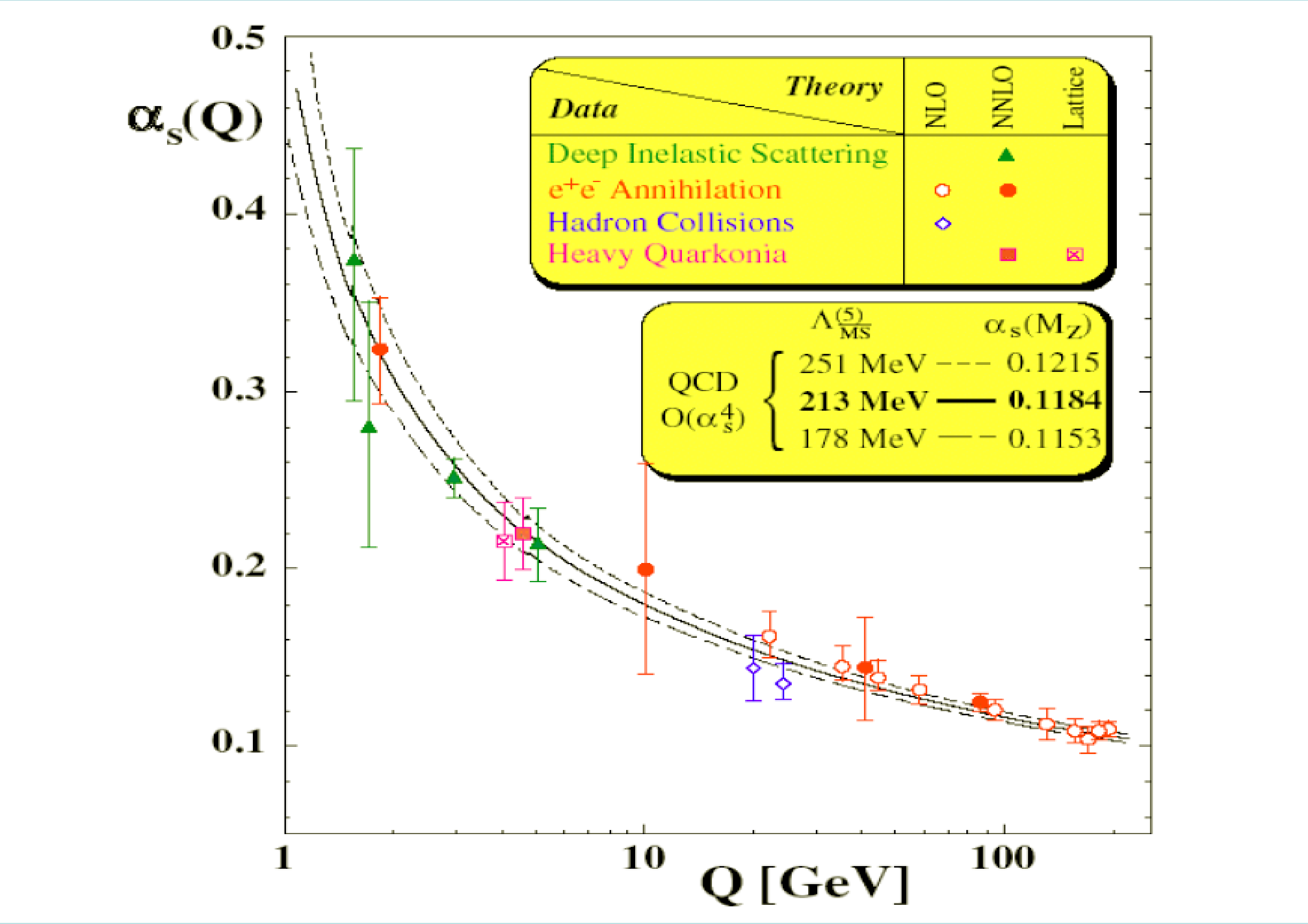
- Conformal infrared behavior

- SU(N) Gauge Theories with N_f Massless Fermions (fundamental and other representations)

- Asymptotically-free (can take lattice spacing to zero) $N_f < N_{af}$

- Vary N_f and study how the infrared behavior changes (chiral symmetry breaking and confinement versus infrared conformal behavior.)

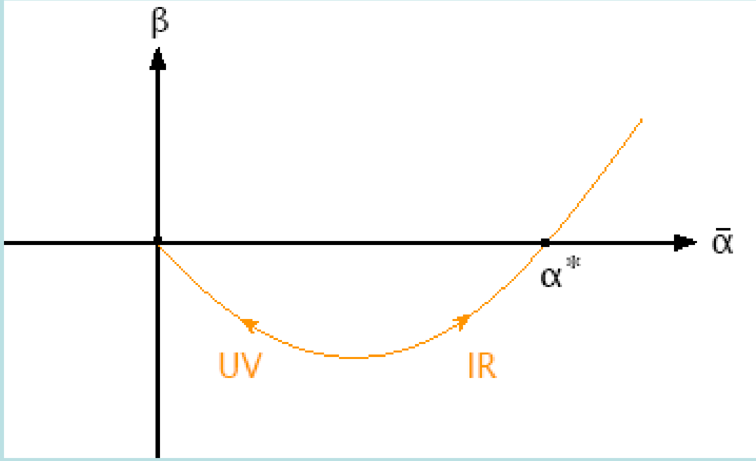
Running



0.5
0.4
0.3
0.2
0.1
$\alpha_s(Q)$
1
10
100
Q [GeV]
Theory
Data
NLO
NNLO
Lattice
Deep Inelastic Scattering
e^+e^- Annihilation
Hadron Collisions
Heavy Quarkonia
$\Lambda^{(5)}_{\overline{MS}}$
$\alpha_s(M_Z)$
QCD
$O(\alpha_s^4)$
251 MeV 0.1215
213 MeV 0.1184
178 MeV 0.1153

Conformal Window

$N_{af} > N_f > N_{fc}$ (Fermion screening)



Perturbative for $N_f \rightarrow N_{af}$

Gross and Wilczek, antiquity

Caswell, 1974 →

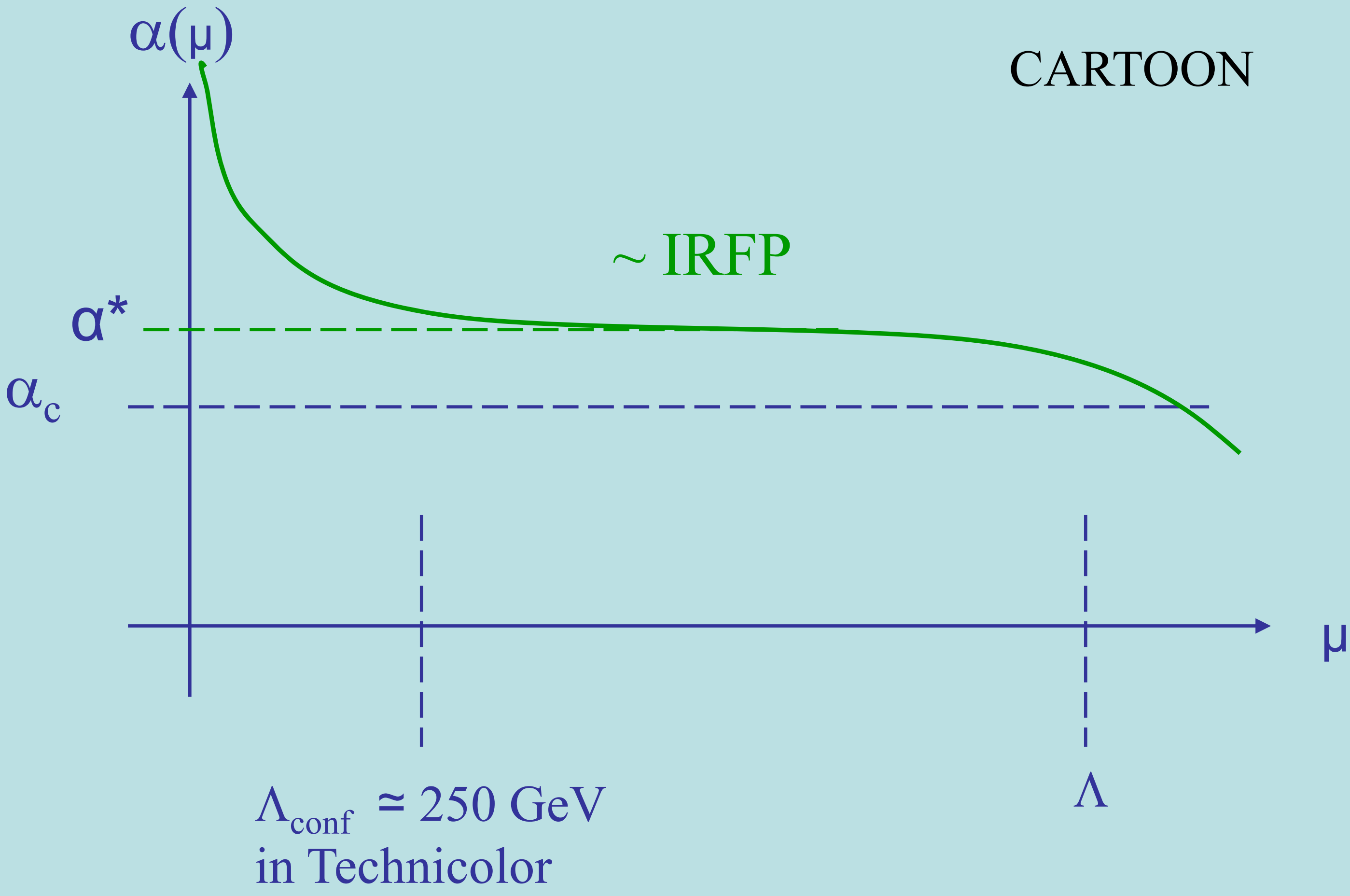
Banks and Zaks, 1982

....

α^* increases as N_f decreases.

N_{fc}: α^* becomes large enough to trigger chiral symmetry breaking and confinement.
(Probably not accessible in perturbation theory)

NEAR-CONFORMAL BEHAVIOR (WALKING)



Questions

1. Value of N_{fc}?

2. Order of the phase transition?

2. Correlation functions and anomalous dimensions inside the Conformal window

3. Below and near the transition:

   Approximate IRFP (Walking)? Condensate enhancement?
   EW precision studies ? S? Implications for the LHC?

TC and ETC CHALLENGES

(1) Precision Electroweak

S, T $(SU(2)_{TC})$

$Z_0 \rightarrow b\bar{b}$

(2) Flavor-Changing Neutral Processes

Walking allows large ETC scales

(3) Complete Theory?

Ingredients to break ETC → TC

Intra-Family mass splittings

Flavor Mixing

CP Violation

Neutrino Mass

POSSIBLE FRAMEWORK

$\begin{pmatrix} u \\ d \end{pmatrix}_L$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$	$\begin{pmatrix} U \\ D \end{pmatrix}_{L4}$	$\begin{pmatrix} U \\ D \end{pmatrix}_{L5}$
u_R	c_R	t_R	U_{R4}	U_{R5}
d_R	s_R	b_R	D_{R4}	D_{R5}
.	.	.	.	.
.	.	.	.	.

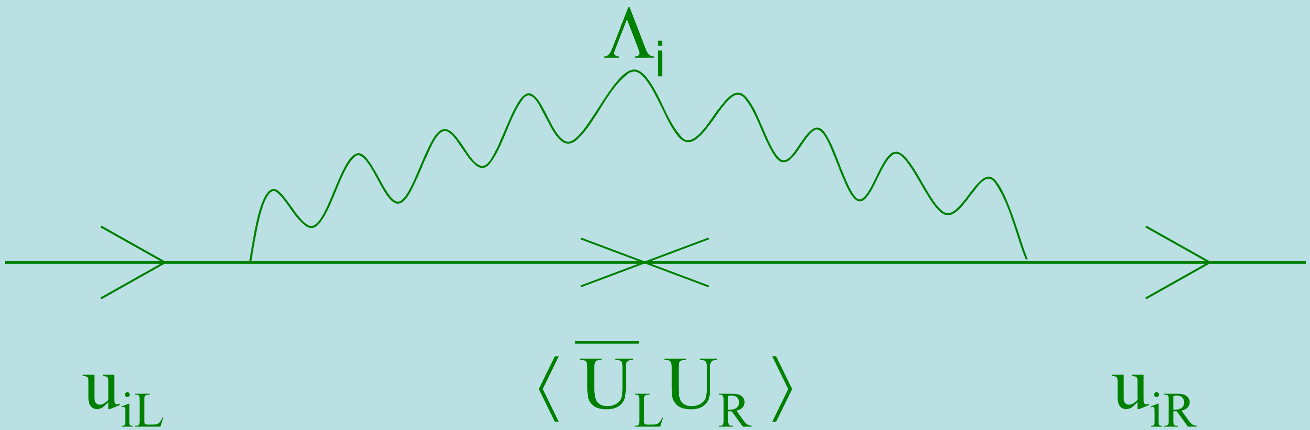
← $SU(N)_{TC}$ →

← $SU(N+3)_{ETC}$ →

Λ_1	Λ_2	Λ_3
$\cong 10^3$Tev	$\cong 10^2$ Tev	$\cong$ Few TeV

→ Baroque Models

FERMION MASS



$$M_{ii}^{(u)} \cong 4\pi \frac{\Lambda_{TC}^{3}}{\Lambda_i^{2}}\,\eta$$

$$\exp\left\{\int \frac{d\mu}{\mu}\,\gamma[\alpha_{TC}(\mu)]\right\} \qquad d=3-\gamma$$

walking ($\cong$ conformal) up to Λ_3 : $\gamma \cong 1$, $\eta \cong \Lambda_3 / \Lambda_i$

N_{fc} in SU(N) QCD

- Degree-of-Freedom Inequality (Cohen, Schmaltz, TA 1999). Fundamental rep:

  $N_{fc} < \; 4\,N[1 - 1/18N^2 + \ldots]$ $\quad (N \geq 3)$

- Gap-Equation Studies, Instantons (1996 ….):

  $N_{fc} \cong 4\,N$

Pioneers:
Maskawa & Nakajima 1974
Kugo & Fukuda 1975

Lattice-Simulation Study of the Extent of the Conformal window in an SU(3) Gauge Theory with Dirac Fermions in the Fundamental Representation

Fleming, Neil, TA

Some Previous Lattice Work with Many Light Fermions

1. Brown et al (Columbia group) P. R. D12, 5655 (1992). $N_f = 8$

2 R. Mahwinney, hep/lat 9701030 (1) $\left(N_f \to 4\right)$
N.P.Proc.Suppl.83:57-66,2000, hep-lat/0001032

3. C. Sui, Flavor dependence of QCD. PhD thesis, Columbia University, 2001. UMI-99-98219

4. Iwasaki et al, Phys. Rev, D69, 014507 (2004)

Focus: Gauge Invariant and Non-Perturbative Definition of the Running Coupling from the Schroedinger Functional of the Gauge Theory

ALPHA Collaboration: Luscher, Sommer, Weisz, Wolff, Bode, Heitger, Simma, …

Transition amplitude from a prescribed state at $t=0$ to one at $t=T$ (Dirichlet BC).($m = 0$)

Using Staggered Fermions as in

U. Heller, Nucl. Phys. B504, 435 (1997)
Miyazaki & Kikukawa

Focus on N_f = multiples of 4:

16: Perturbative IRFP

12:	IRFP "expected", Simulate
8 :	IRFP uncertain , Simulate

4 : Confinement, ChSB

SF Running Coupling

To remove the O(a) bulk lattice artifact

$$\frac{1}{\bar{g}^2(L)} = \frac{1}{2}\left[\frac{1}{\bar{g}^2(L, L-a)} + \frac{1}{\bar{g}^2(L, L+a)}\right]$$

Depends on only one large scale L!

At three loops

$N_f = 16$ IRFP at $g^{*2}_{SF} = 0.47$ $(g^{*2}_{SF}/4\pi \approx .04)$

$N_f = 12$ IRFP at $g^{*2}_{SF} = 5.18$ $(g^{*2}_{SF}/4\pi \approx 0.4)$

$N_f \leq 8$ No perturbative IRFP

Lattice Simulations

MILC Code (Heller)

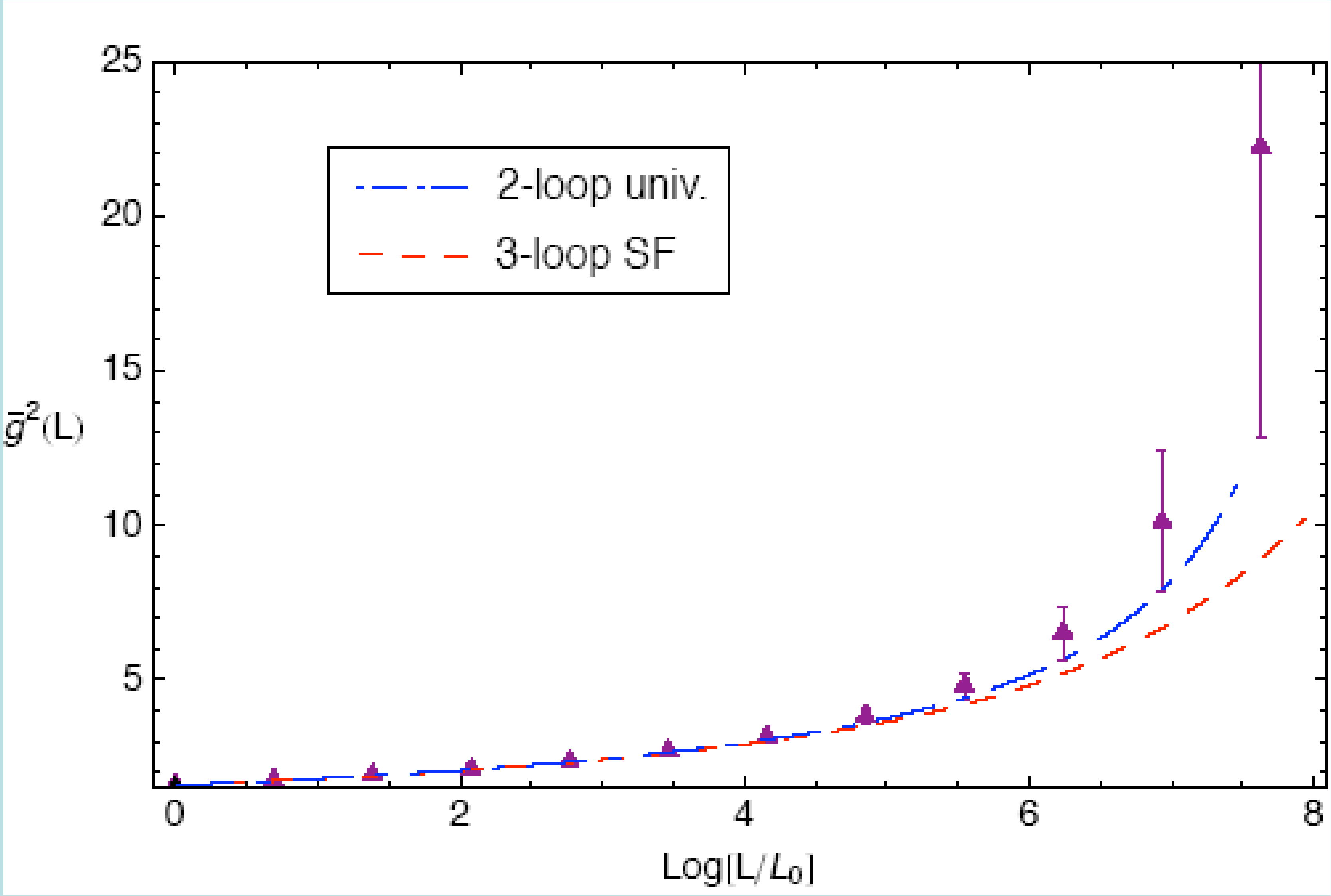
Staggered Fermions

$$N_f = 8, 12$$

Range of Lattice Couplings g_0^2 (= $6/\beta$) and Lattice Sizes $L/a \rightarrow 20$

O(a) Lattice Artifacts due to Dirichlet Boundary Conditions

N_f = 8 Continuum Running



N_f = 8 Features

No evidence for IRFP or even inflection point.

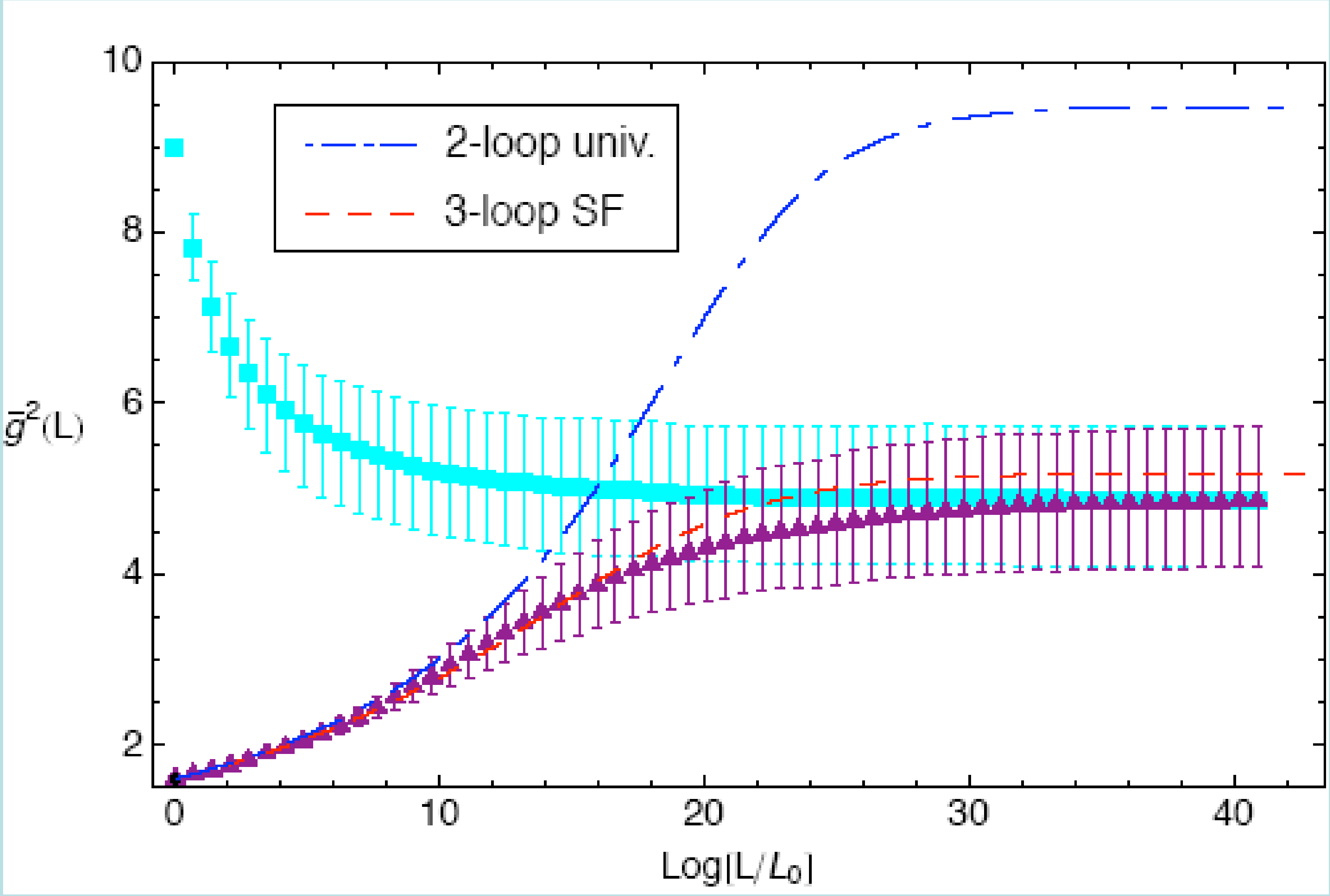
.

Exceeds rough estimate $\left(\alpha_c^* / \pi \approx 1/4\right)$ of strength required to break chiral symmetry, and confine.

Rate of growth exceeds 3 loop perturbation theory.

Behavior similar to quenched theory [ALPHA N.P. Proc. Suppl. 106, 859 (2002)] and N_f=2 theory [ALPHA, N.P. B713, 378 (2005)], but slower growth as expected.

N_f = 12 Continuum Running



Conclusions

1. Lattice evidence that for an SU(3) gauge theory with N_f Dirac fermions in the fundamental representation
   $8 < N_{fc} < 12$

2. N_f=12: Relatively weak IRFP

3. N_f=8: Confinement and chiral symmetry breaking.

Employing the Schroedinger-functional running coupling defined at the box boundary L

Current $N_f = 12$ Activity

Summary of lattice results at $N_f = 12$ (3-color fundamental)

Group	Fermion action	Gauge action	Method	Conformal?	Refs (arXiv)
Fleming, Neil, TA	staggered	unimproved (plaquette)	SF coupling	Yes	0712.0609 0901.3766
Jin & Mawhinney	staggered	improved (DBW2)	Spectrum	No	0910.3216
Hasenfratz	improved staggered (stout)	unimproved (plaquette)	MCRG	Yes	0911.0646 priv. comm.
Kuti, Holland, Fodor, et al.	improved staggered (stout)	improved (Symanzik)	Spectrum	No	0911.2463
Deuzeman, Lombardo, Pallante	improved staggered (Naik)	improved (Symanzik)	Thermal trans.	Yes	0904.4662

Things to Do

1. Other N_f values such as N_f =10.

   Yamada et al

2. Other gauge groups and representation assignments for the fermions

   Sannino, del Debbio, DeGrand, Shamir, Svetitsky; Hietenan et al, Sinclair, Kogut, Catterall. ……..

3. Examine physical quantities such as the static potential (Wilson loop) and correlation functions.

   Bilgici et al arXiv:0902.3768

5. Examine chiral symmetry breaking directly:

$\langle\bar{\psi}\psi\rangle / F^3$ for $N_f = 2,4,6,8,\ldots\ldots$ LSD

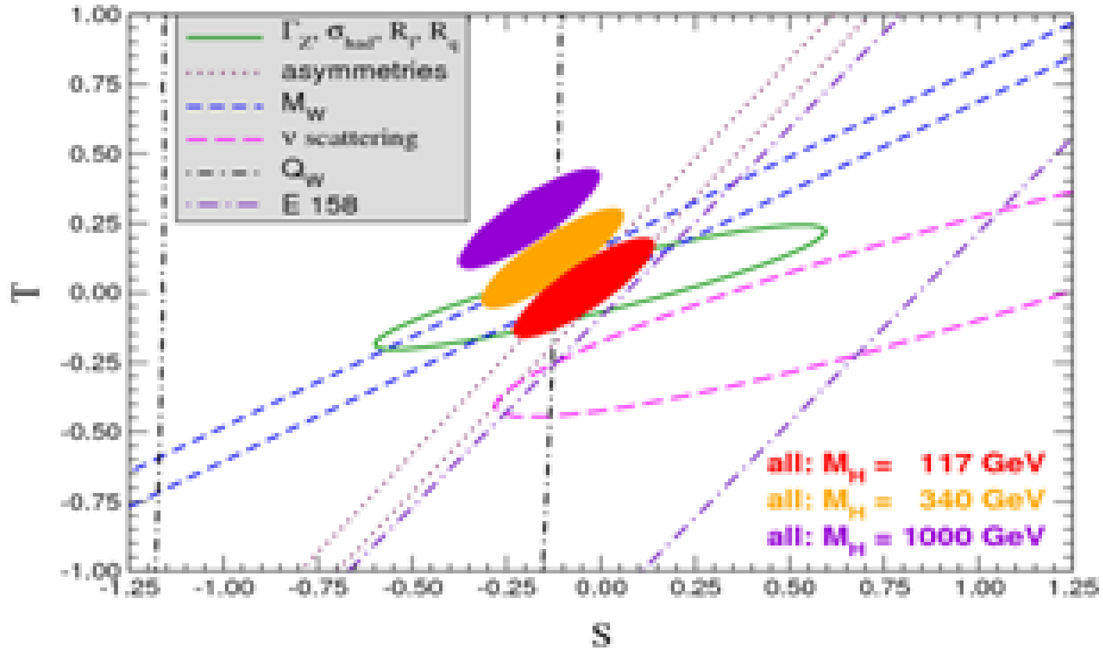
Enhancement? Parity Doubling?

6. Apply to BSM Physics. Is S naturally small as $N_f \rightarrow N_{fc}$ due to approximate parity doubling?

LSD

$$S\left(m_{H,ref}\right) = 4\int_0^\infty \frac{ds}{s}\left\{\left[\operatorname{Im}\Pi_{VV}(s) - \operatorname{Im}\Pi_{AA}(s)\right] - \frac{1}{48\pi}\left[1 - \left(1 - \frac{m_{H,ref}}{s}\right)^3 \theta\left(s - m_{H,ref}^2\right)\right]\right\}$$

S and T



$S_{free} = (1/2\pi)(N/3)(N_f/2)$ $\qquad$ $S_{QCD} = 0.3\ (N/3)(N_f/2)$

Condensate Enhancement LSD

Walking:

As the conformal window is approached ($N_f \rightarrow N_{fc}$),
$\langle \bar{\psi} \psi \rangle$ is enhanced relative to its nominal value $4\pi F^3$.
Ultimately $O(4\pi F^2 \Lambda)$ where Λ is the relevant UV cutoff

Current LSD Program:

Search for enhancement of $\langle \bar{\psi} \psi \rangle / F^3$ by starting at $N_f = 2$,
and then $N_f = 6$. (Creeping Toward the Conformal Window)
($\Lambda = a^{-1}$)

Some Details

- Domain-wall fermions, Iwasaki improved action
- USQCD: Chroma, CPS

- 32^3 x 64 lattice ($L_s = 16$)
- $m_f = .005, .01, .015, .02, .025$, $m = m_f + m_{res}$

- Simulate: M^2_m , F_m, $\langle\bar{\psi}\,\psi\rangle_m$, $M_{\rho,m}$, $r_{o,m}$

$M_m L > 4$

Extrapolate to m=0 with Chiral Perturbation Theory

- $M^2_m = 2m \langle\bar{\psi}\,\psi\rangle / F^2 \{1 + zm\,[\alpha_{M1} + (1/N_f)\,\log(zm)] + \ldots\}$

- $F_m = F\{ 1 + zm\,[\alpha_{F1} - (N_f/2)\,\log(zm)] + \ldots\}$

- $\langle\bar{\psi}\,\psi\rangle_m = \langle\bar{\psi}\,\psi\rangle \{1 + zm\,[\alpha_{C1} - ((N_f^2 - 1)/N_f)\,\log(zm)] + \ldots\}$

$$z \equiv 2 \langle\bar{\psi}\,\psi\rangle / (4\pi)^2 F^4$$

Chiral Perturbation Theory for the rho and a1 Masses

$$M_{\rho,m} = M_\rho \{ 1 + \alpha_{R1}\, zm + \alpha_{R3/2}\, (zm)^{3/2} + \ldots \}$$

$$M_{a1,m} = M_{a1} \{ 1 + \alpha_{A1}\, zm + \alpha_{A3/2}\, (zm)^{3/2} + \ldots \}$$

$$r_{0m} = \ldots\ldots.$$

Program

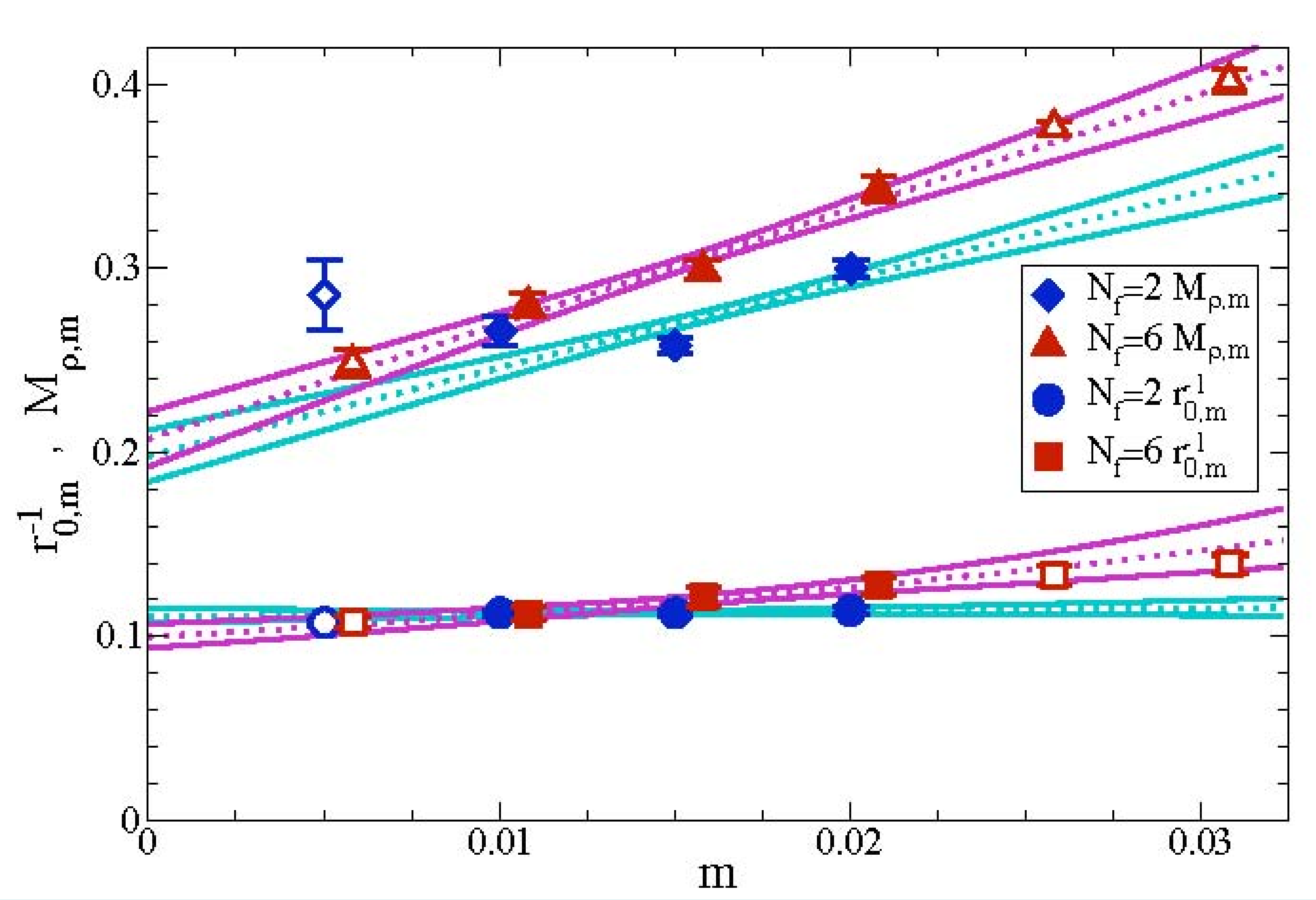
- Choose $\beta \equiv 6/g_0^2$ for $N_f = 2$, and then for $N_f = 6$, to keep M_ρ (and $4\pi F$) the same in lattice units for the two cases. Compare the $N_f = 2$ results for $<\bar{\psi}\,\psi>/4\pi F^3$, M_ρ/F , M_{a1}/F to the QCD phenomenological Values.

- For $N_f = 6$, compute

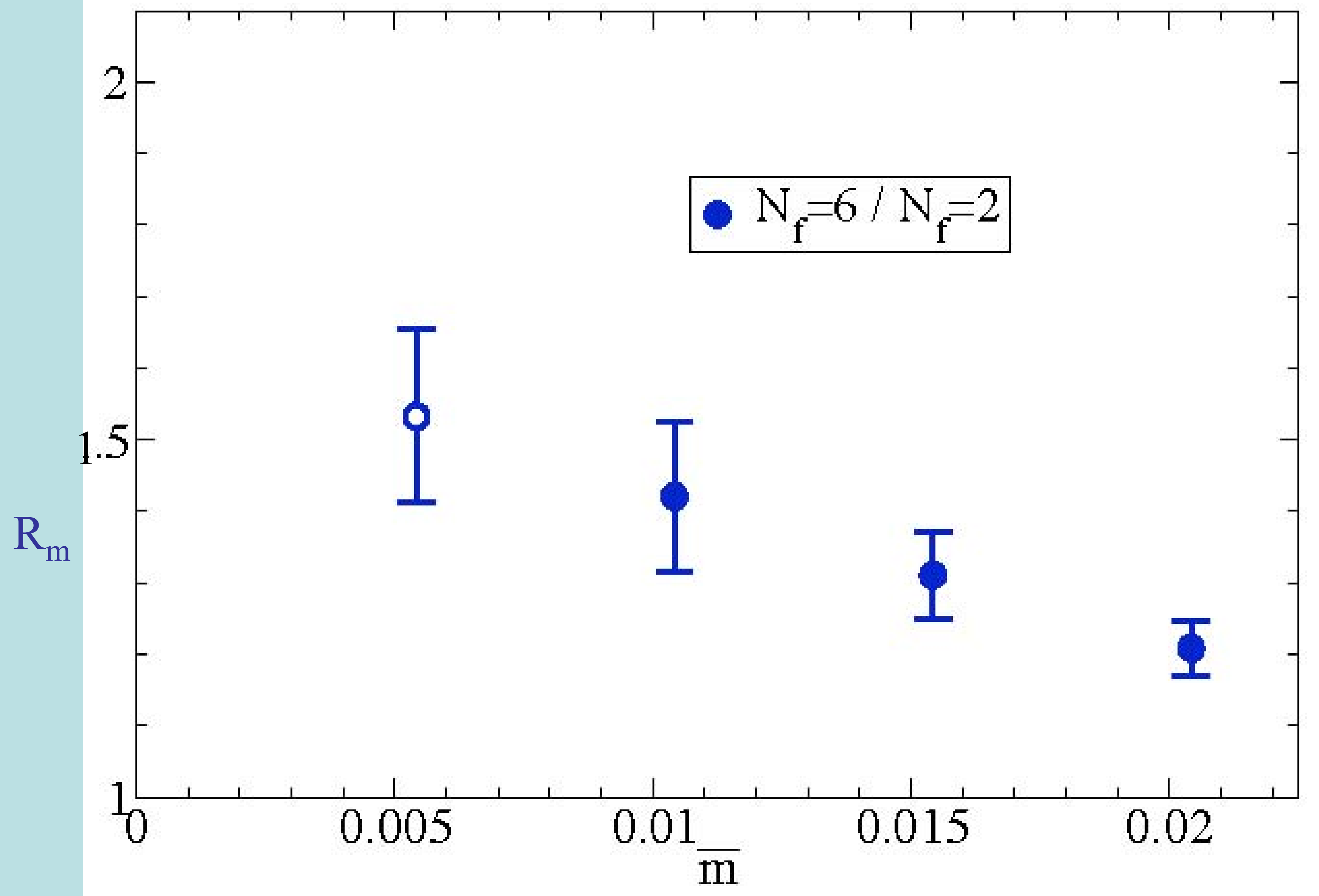
  $<\bar{\psi}\,\psi>/4\pi F^3$ Enhancement?

  $(M_{a1} - M_\rho) / M_\rho$ → Parity doubling?

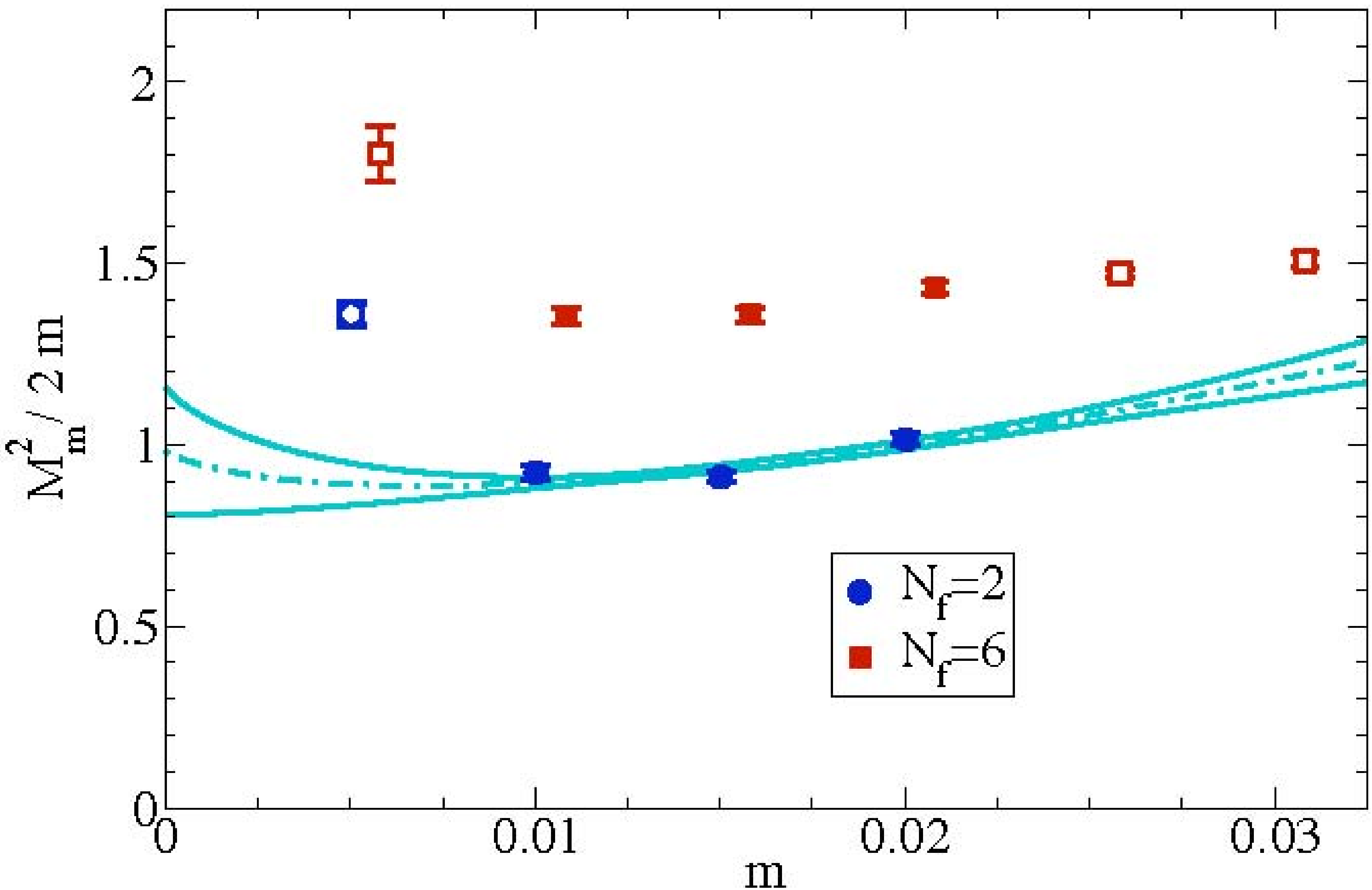
- Future: Estimate S. Then on to $N_f = 8, 10, \ldots N_{fc}$ (and into the conformal window).



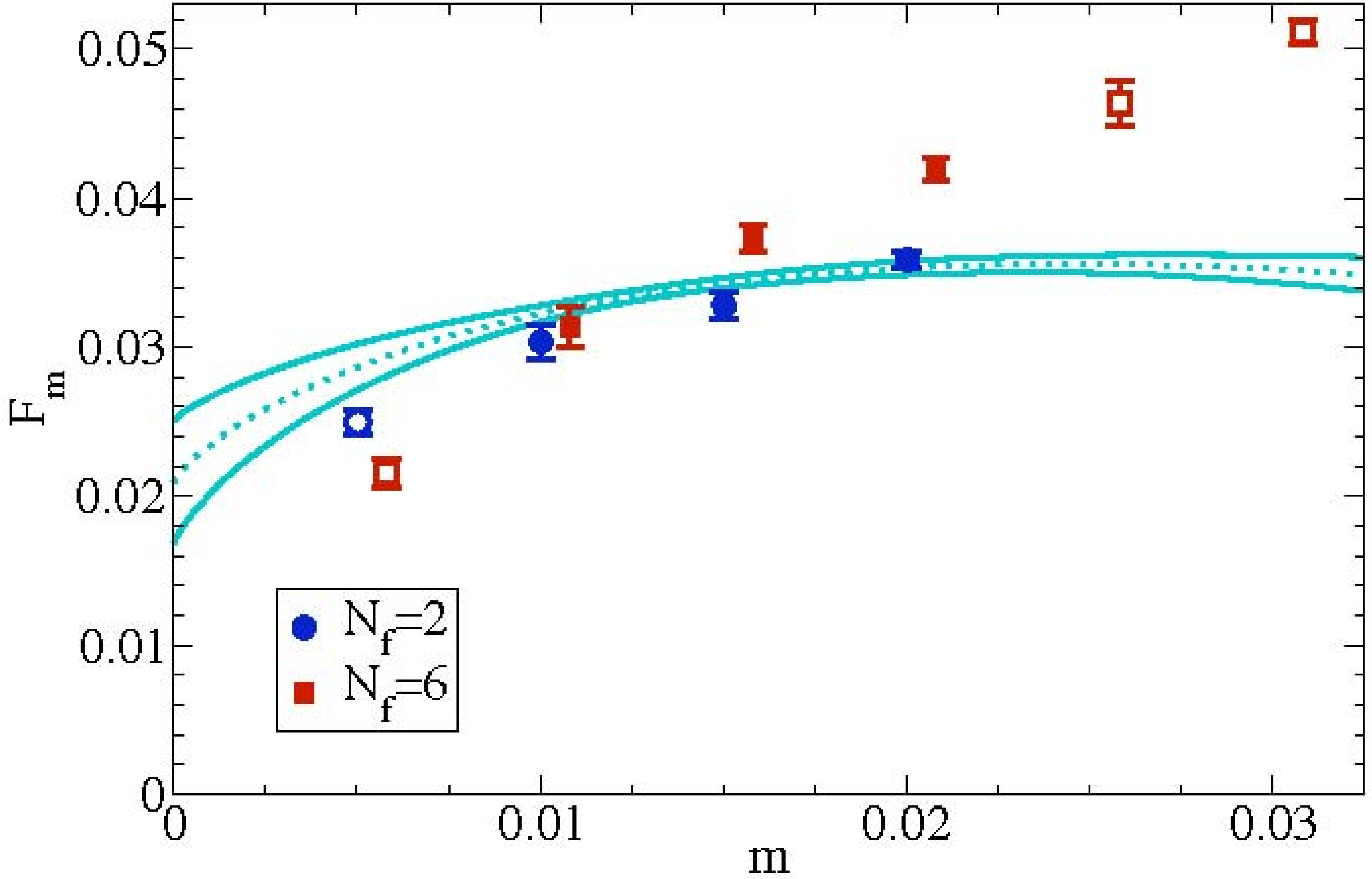
0.4
0.3
0.2
0.1
0
0
0.01
0.02
0.03
m
$r_{0,m}^{-1}$, $M_{\rho,m}$
N_f=2 $M_{\rho,m}$
N_f=6 $M_{\rho,m}$
N_f=2 $r_{0,m}^{-1}$
N_f=6 $r_{0,m}^{-1}$



$R_m = [M^2_m/2mF_m]_{6f} / [M^2_m/2mF_m]_{2f}$



$M_m^2 / 2m$
2
1.5
1
0.5
0
0
0.01
0.02
0.03
m
N_f=2
N_f=6



0.05
0.04
0.03
0.02
0.01
0
F_m
0
0.01
0.02
0.03
m
N_f=2
N_f=6

$N_f = 2$

- Chiral perturbation theory extrapolation:

$$\langle \bar{\psi} \psi \rangle / F^3 = 47.1\ (17.6)$$

QCD Experimental Value: (renormalized to our lattice scheme - Aoki et al hep-lat/0206013)

$$\langle \bar{\psi} \psi \rangle / F^3 = 36.2\ (6.5)$$

$N_f = 6$

Linear Extrapolation →

Very Conservative Lower Bound on $\langle\bar{\psi}\,\psi\rangle / F^2$

Very Conservative Upper Bound on F

Thus

$$\langle\bar{\psi}\,\psi\rangle / F^3 \geq 60.0\ (8.0)$$

Conclusion

The ratio $<\bar{\psi}\psi>/F^3$ in an SU(3) gauge theory with N_f massless Dirac fermions in the fundamental representation, with the condensate defined by a lattice cutoff fixed in physical units, is enhanced when N_f is increased from 2 to 6 -- by at least 50% from inspection of the simulation results. Even a very conservative lower bound indicates a substantial increase. An enhancement of less than 50% would require a significant and hard-to-explain downturn in R_m. We expect the enhancement seen here, arising dominantly at the lattice scale, to be qualitatively the same as with a continuum cutoff from the onset of new physics.

Summary

1. The lattice is beginning to teach us things about theories that might describe physics beyond the standard model.

2. Work in this area is accelerating.

3. Very interesting related analytic studies

4. Lots of work to do.