

TeV Physics and Conformality



Cape Town February 1, 2010

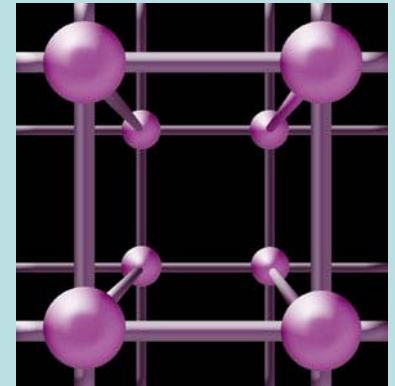
1. Collaboration with George Fleming and Ethan Neil

1) arXiv:0712.0609, PRL 100, 171607, 2008

2) arXiv:0901.3766 PR D79, 076010, 2009

2. **LSD** collaboration J. C. Osborn, R. Babich, R. C. Brower,
M. A. Clark, C. Rebbi, D. Schaich, M. Cheng, T. Luu,
R. Soltz, P. M. Vranas, T. Appelquist, G. T. Fleming,
E. T. Neil, Meifeng Lin

arXiv:0910.2224 → PRL



New Strong Dynamics



Electroweak breaking

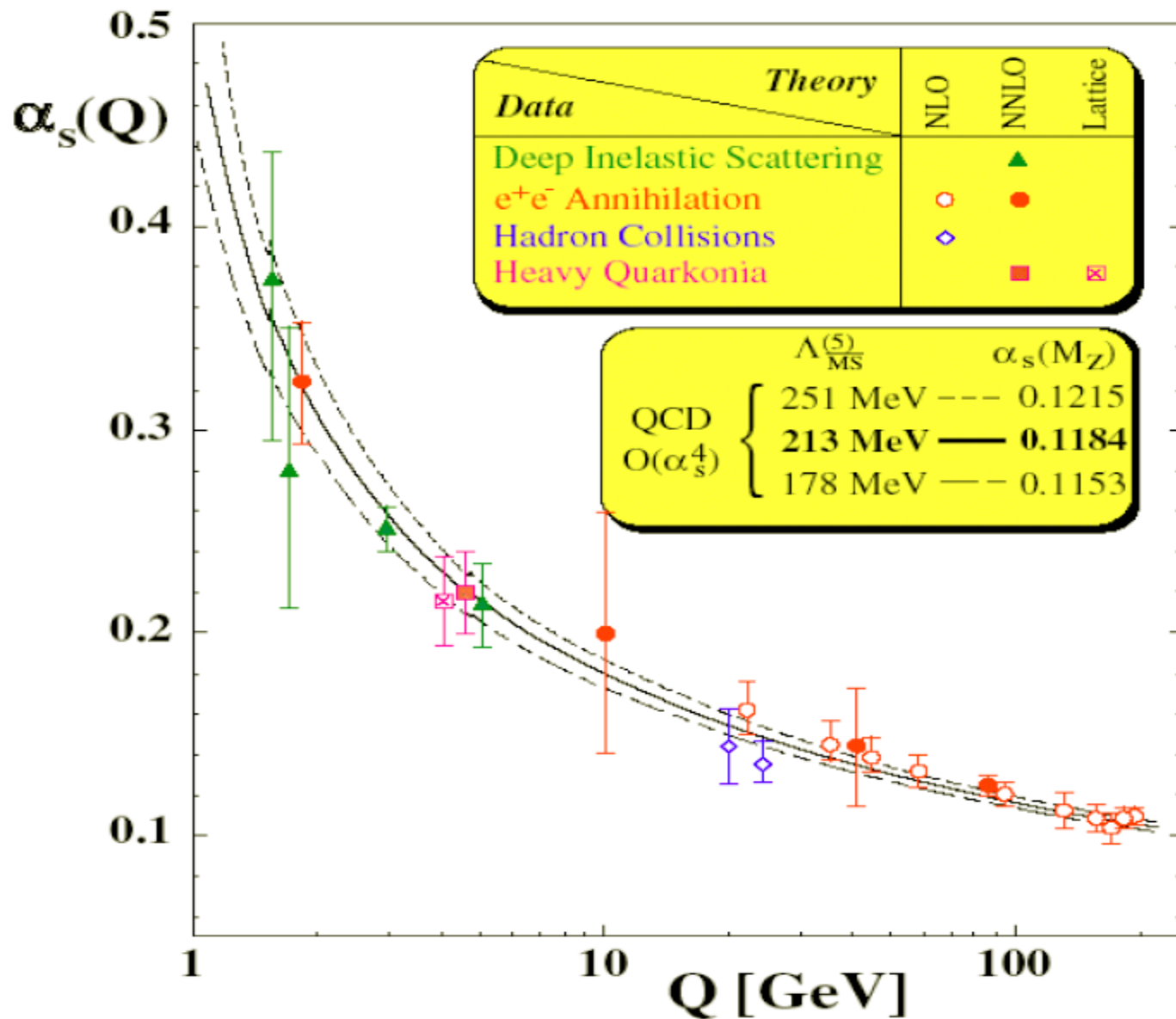
- Technicolor, Higgsless theories, ADS/CFT approaches
- Near-conformal infrared behavior: walking technicolor

New, SM-singlet Sector

- Conformal infrared behavior

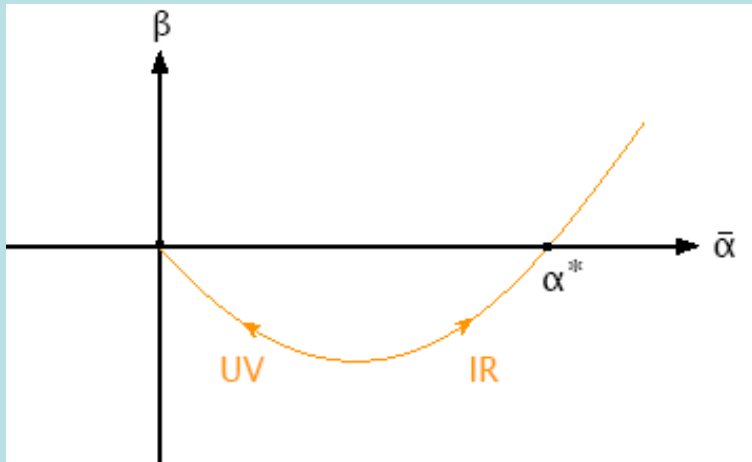
- SU(N) Gauge Theories with N_f Massless Fermions (fundamental and other representations)
- Asymptotically-free (can take lattice spacing to zero) $N_f < N_{af}$
- Vary N_f and study how the infrared behavior changes (chiral symmetry breaking and confinement versus infrared conformal behavior.)

Running



Conformal Window

$$N_{af} > N_f > N_{fc} \quad (\text{Fermion screening})$$



Perturbative for $N_f \rightarrow N_{af}$

Gross and Wilczek, antiquity

Caswell, 1974 →

Banks and Zaks, 1982

.....

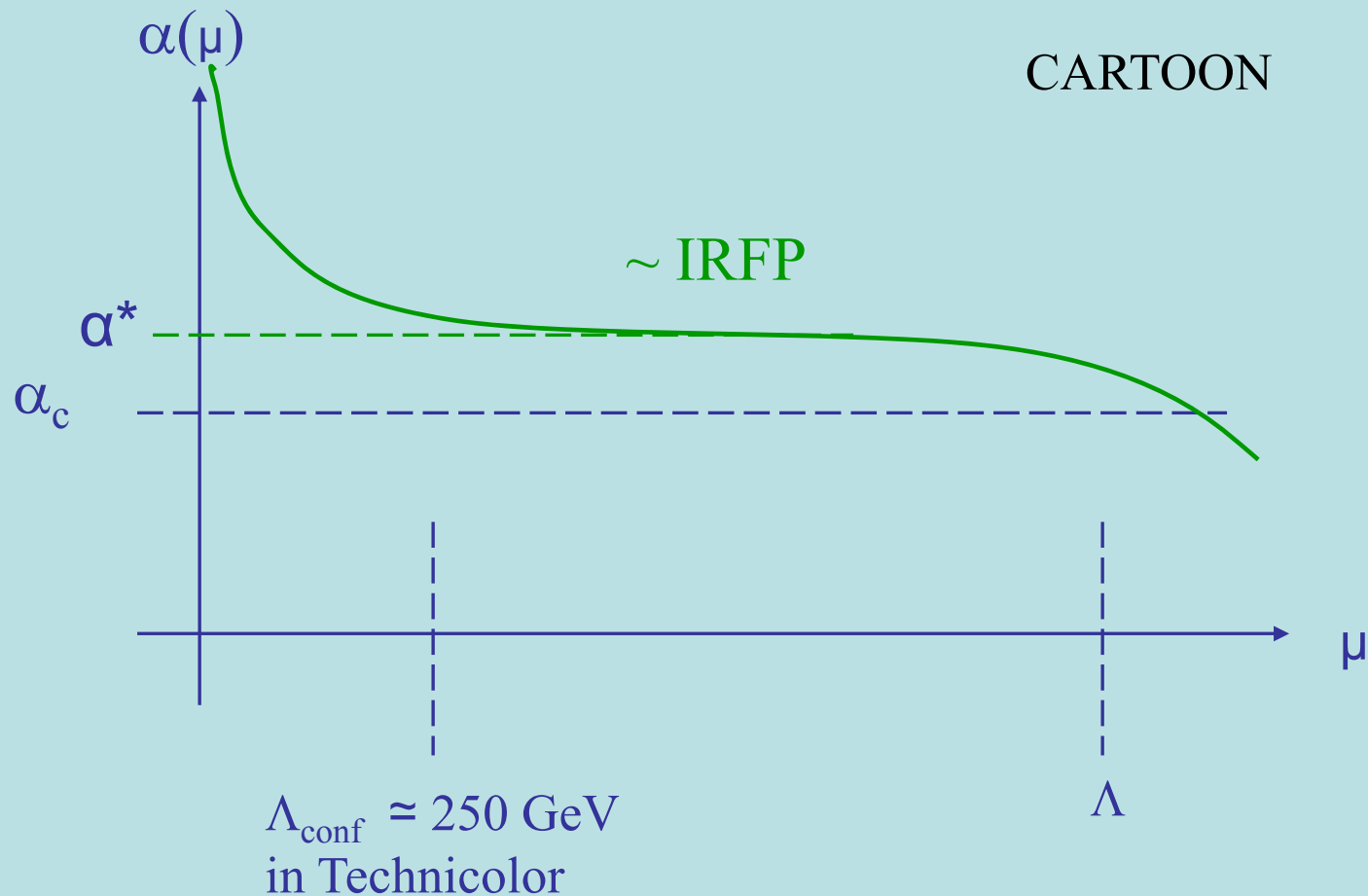


α^* increases as N_f decreases.

N_{fc} : α^* becomes large enough to trigger chiral symmetry breaking and confinement.

(Probably not accessible in perturbation theory)

NEAR-CONFORMAL BEHAVIOR (WALKING)



Questions

1. Value of N_{fc} ?
2. Order of the phase transition?
2. Correlation functions and anomalous dimensions inside the Conformal window
3. Below and near the transition:
 - Approximate IRFP (Walking)? Condensate enhancement?
 - EW precision studies ? S? Implications for the LHC?

TC and ETC CHALLENGES

(1) Precision Electroweak

$$S, T \quad (SU(2)_{TC})$$
$$Z_0 \rightarrow b\bar{b}$$



(2) Flavor-Changing Neutral Processes

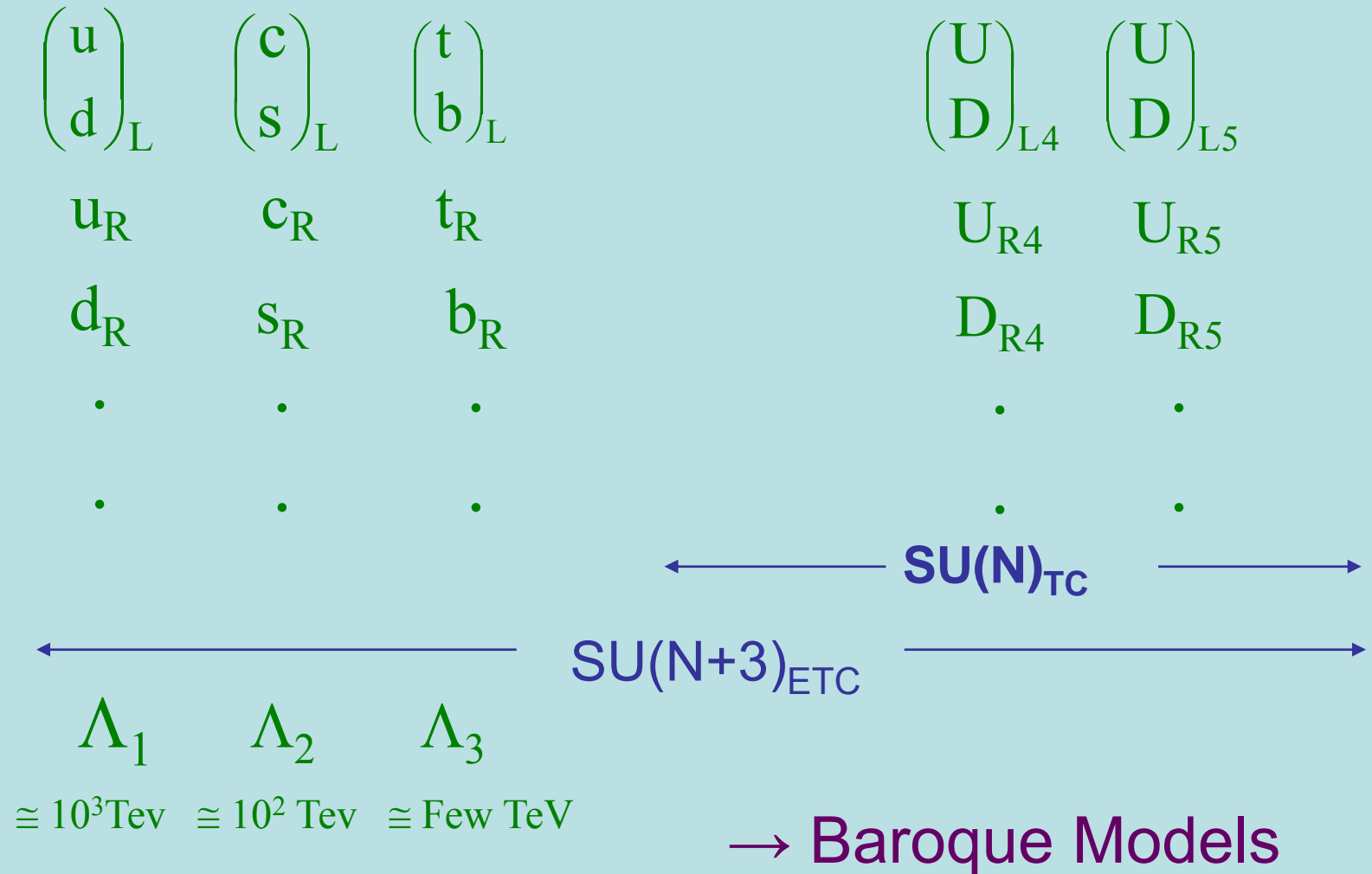
Walking allows large ETC scales

(3) Complete Theory?

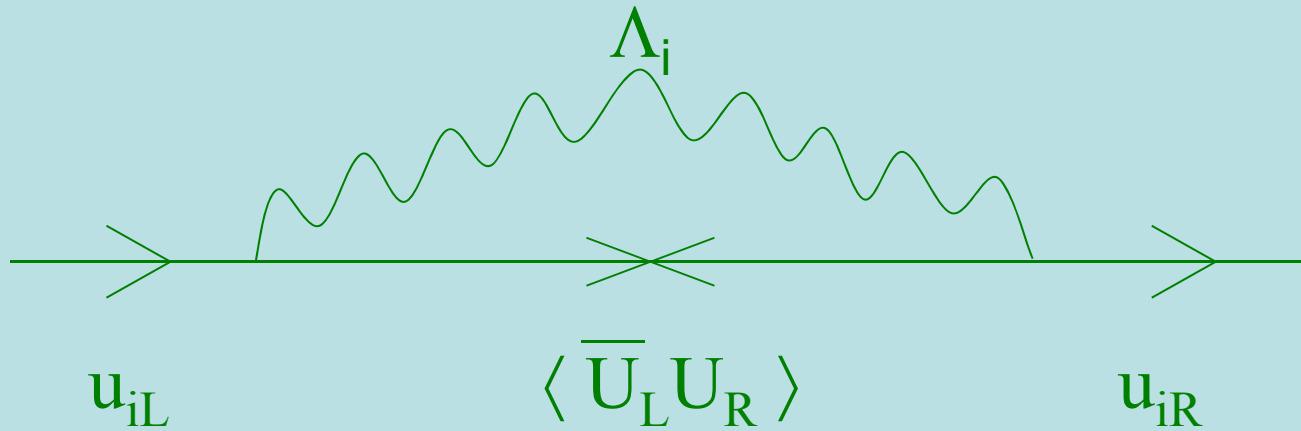
Ingredients to break ETC \rightarrow TC

- Intra-Family mass splittings
- Flavor Mixing
- CP Violation
- Neutrino Mass

POSSIBLE FRAMEWORK



FERMION MASS



$$M_{ii}^{(u)} \cong 4\pi \frac{\Lambda_{TC}^3}{\Lambda_i^2} \eta$$

$$\exp \left\{ \int_{\mu}^{\Lambda_{TC}} \frac{d\mu}{\mu} \gamma[\alpha_{TC}(\mu)] \right\} \quad d=3-\gamma$$

walking (\cong conformal) up to Λ_3 : $\gamma \cong 1$, $\eta \cong \Lambda_3 / \Lambda_i$

N_{fc} in $SU(N)$ QCD

- Degree-of-Freedom Inequality (Cohen, Schmaltz, TA 1999). Fundamental rep:

$$N_{fc} < 4 N [1 - 1/18N^2 + \dots] \quad (N \geq 3)$$

- Gap-Equation Studies, Instantons (1996):

$$N_{fc} \cong 4 N$$

Pioneers:

Maskawa & Nakajima 1974

Kugo & Fukuda 1975

Lattice-Simulation Study of the Extent of the Conformal window in an SU(3) Gauge Theory with Dirac Fermions in the Fundamental Representation



Fleming, Neil, TA

Some Previous Lattice Work with Many Light Fermions

1. Brown et al (Columbia group) P. R. D12, 5655 (1992).
 $N_f = 8$
2. R. Mahwinney, hep/lat 9701030 (1) ($N_f \rightarrow 4$)
N.P.Proc.Suppl.83:57-66,2000, hep-lat/0001032
3. C. Sui, Flavor dependence of QCD. PhD thesis,
Columbia University, 2001. UMI-99-98219
4. Iwasaki et al, Phys. Rev, D69, 014507 (2004)

Focus: Gauge Invariant and Non-Perturbative Definition of the Running Coupling from the Schroedinger Functional of the Gauge Theory

ALPHA Collaboration: Luscher, Sommer, Weisz, Wolff, Bode, Heitger, Simma, ...

Transition amplitude from a prescribed state at $t=0$ to one at $t=T$ (Dirichlet BC). ($m = 0$)

Using Staggered Fermions as in

U. Heller, Nucl. Phys. B504, 435 (1997)
Miyazaki & Kikukawa

Focus on $N_f =$ multiples of 4:

16: Perturbative IRFP

12: IRFP “expected”, Simulate

8 : IRFP uncertain , Simulate

4 : Confinement, ChSB

SF Running Coupling

To remove the $O(a)$ bulk lattice artifact

$$\frac{1}{\bar{g}^2(L)} = \frac{1}{2} \left[\frac{1}{\bar{g}^2(L, L-a)} + \frac{1}{\bar{g}^2(L, L+a)} \right]$$

Depends on only one large scale L !

At three loops

$$N_f = 16 \quad \text{IRFP at} \quad g_{SF}^{*2} = 0.47 \quad (g_{SF}^{*2}/4\pi \approx .04)$$

$$N_f = 12 \quad \text{IRFP at} \quad g_{SF}^{*2} = 5.18 \quad (g_{SF}^{*2}/4\pi \approx 0.4)$$

$$N_f \leq 8 \quad \text{No perturbative IRFP}$$

Lattice Simulations

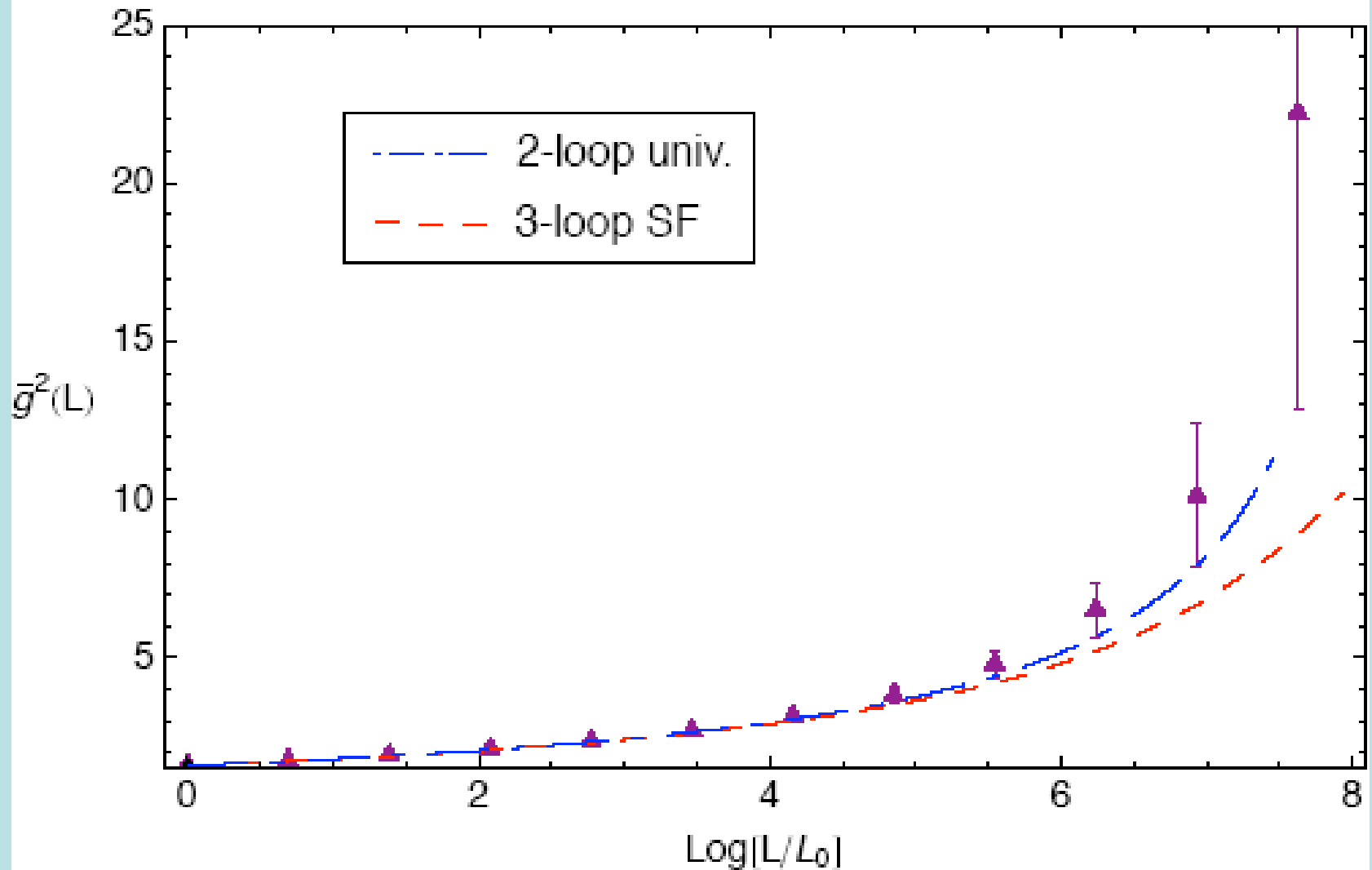
MILC Code (Heller)
Staggered Fermions

$$N_f = 8, 12$$

Range of Lattice Couplings $g_0^2 (= 6/\beta)$ and Lattice
Sizes $L/a \rightarrow 20$

$O(a)$ Lattice Artifacts due to Dirichlet Boundary
Conditions

$N_f = 8$ Continuum Running



$N_f = 8$ Features

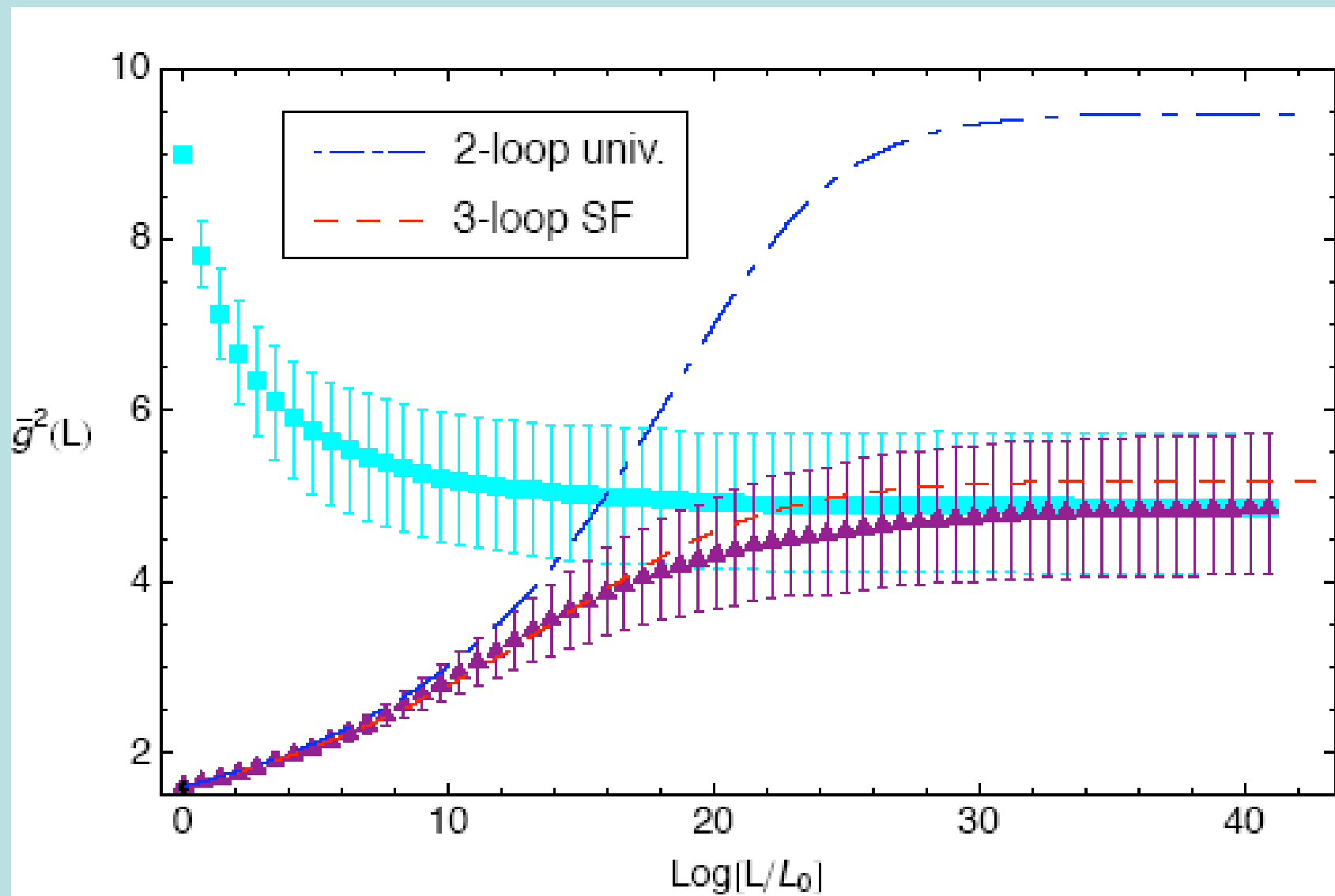
No evidence for IRFP or even inflection point.

Exceeds rough estimate $(\alpha_c^*/\pi \approx 1/4)$ of strength required to break chiral symmetry, and confine.

Rate of growth exceeds 3 loop perturbation theory.

Behavior similar to quenched theory [ALPHA N.P. Proc. Suppl. 106, 859 (2002)] and $N_f=2$ theory [ALPHA, N.P. B713, 378 (2005)], but slower growth as expected.

$N_f = 12$ Continuum Running



Conclusions

1. Lattice evidence that for an SU(3) gauge theory with N_f Dirac fermions in the fundamental representation $8 < N_{fc} < 12$
2. $N_f=12$: Relatively weak IRFP
3. $N_f=8$: Confinement and chiral symmetry breaking.

Employing the Schroedinger-functional running coupling defined at the box boundary L

Current $N_f = 12$ Activity

Summary of lattice results at $N_f = 12$ (3-color fundamental)

Group	Fermion action	Gauge action	Method	Conformal?	Refs (arXiv)
Fleming, Neil, TA	staggered	unimproved (plaquette)	SF coupling	Yes	0712.0609 0901.3766
Jin & Mawhinney	staggered	improved (DBW2)	Spectrum	No	0910.3216
Hasenfratz	improved staggered (stout)	unimproved (plaquette)	MCRG	Yes	0911.0646 priv. comm.
Kuti, Holland, Fodor, et al.	improved staggered (stout)	improved (Symanzik)	Spectrum	No	0911.2463
Deuzeman, Lombardo, Pallante	improved staggered (Naik)	improved (Symanzik)	Thermal trans.	Yes	0904.4662

Things to Do

1. Other N_f values such as $N_f=10$.

Yamada et al

2. Other gauge groups and representation assignments for the fermions

Sannino, del Debbio, DeGrand, Shamir, Svetitsky; Hietanen et al, Sinclair, Kogut, Catterall.

3. Examine physical quantities such as the static potential (Wilson loop) and correlation functions.

Bilgici et al [arXiv:0902.3768](https://arxiv.org/abs/0902.3768)

5. Examine chiral symmetry breaking directly:

$\langle \bar{\psi} \psi \rangle / F^3$ for $N_f = 2, 4, 6, 8, \dots$ LSD

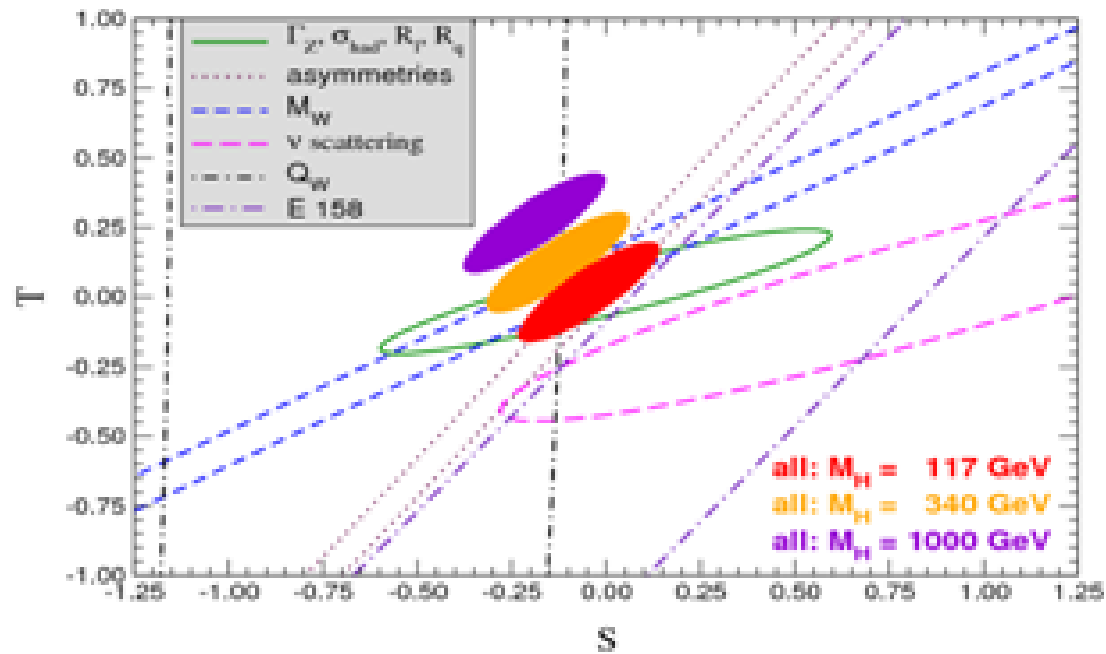
Enhancement? Parity Doubling?

6. Apply to BSM Physics. Is S naturally small as $N_f \rightarrow N_{fc}$ due to approximate parity doubling?

LSD

$$S(m_{H,ref}) = 4 \int_0^{\infty} \frac{ds}{s} \left\{ \left[\text{Im} \Pi_{VV}(s) - \text{Im} \Pi_{AA}(s) \right] - \frac{1}{48\pi} \left[1 - \left(1 - \frac{m_{H,ref}^2}{s} \right)^3 \theta(s - m_{H,ref}^2) \right] \right\}$$

S and T



$$S_{\text{free}} = (1/2\pi)(N/3)(N_f/2)$$

$$S_{\text{QCD}} = 0.3 (N/3)(N_f/2)$$

Condensate Enhancement LSD

Walking:

As the conformal window is approached ($N_f \rightarrow N_{fc}$),
 $\langle \bar{\psi} \psi \rangle$ is enhanced relative to its nominal value $4\pi F^3$.
Ultimately $O(4\pi F^2 \Lambda)$ where Λ is the relevant UV cutoff

Current LSD Program:

Search for enhancement of $\langle \bar{\psi} \psi \rangle / F^3$ by starting at $N_f = 2$,
and then $N_f = 6$. (Creeping Toward the Conformal Window)
($\Lambda = a^{-1}$)

Some Details

- Domain-wall fermions, Iwasaki improved action
- USQCD: Chroma, CPS
- $32^3 \times 64$ lattice ($L_s = 16$)
- $m_f = .005, .01, .015, .02, .025$, $m = m_f + m_{res}$
- Simulate: M_m^2 , F_m , $\langle \bar{\psi} \psi \rangle_m$, $M_{\rho,m}$, $r_{o,m}$

$$M_m L > 4$$

Extrapolate to $m=0$ with Chiral Perturbation Theory

- $M_m^2 = 2m \langle \bar{\psi} \psi \rangle / F^2 \{ 1 + zm [\alpha_{M1} + (1/N_f) \log(zm)] + \dots \}$
- $F_m = F \{ 1 + zm [\alpha_{F1} - (N_f/2) \log(zm)] + \dots \}$
- $\langle \bar{\psi} \psi \rangle_m = \langle \bar{\psi} \psi \rangle \{ 1 + zm [\alpha_{C1} - ((N_f^2 - 1)/N_f) \log(zm)] + \dots \}$

$$z \equiv 2 \langle \bar{\psi} \psi \rangle / (4\pi)^2 F^4$$

Chiral Perturbation Theory for the rho and a1 Masses

$$M_{\rho,m} = M_{\rho} \{ 1 + \alpha_{R1} zm + \alpha_{R3/2} (zm)^{3/2} + \dots \}$$

$$M_{a1,m} = M_{a1} \{ 1 + \alpha_{A1} zm + \alpha_{A3/2} (zm)^{3/2} + \dots \}$$

$$r_{0m} = \dots\dots\dots$$

Program

- Choose $\beta \equiv 6/g_0^2$ for $N_f = 2$, and then for $N_f = 6$, to keep M_ρ (and $4\pi F$) the same in lattice units for the two cases.

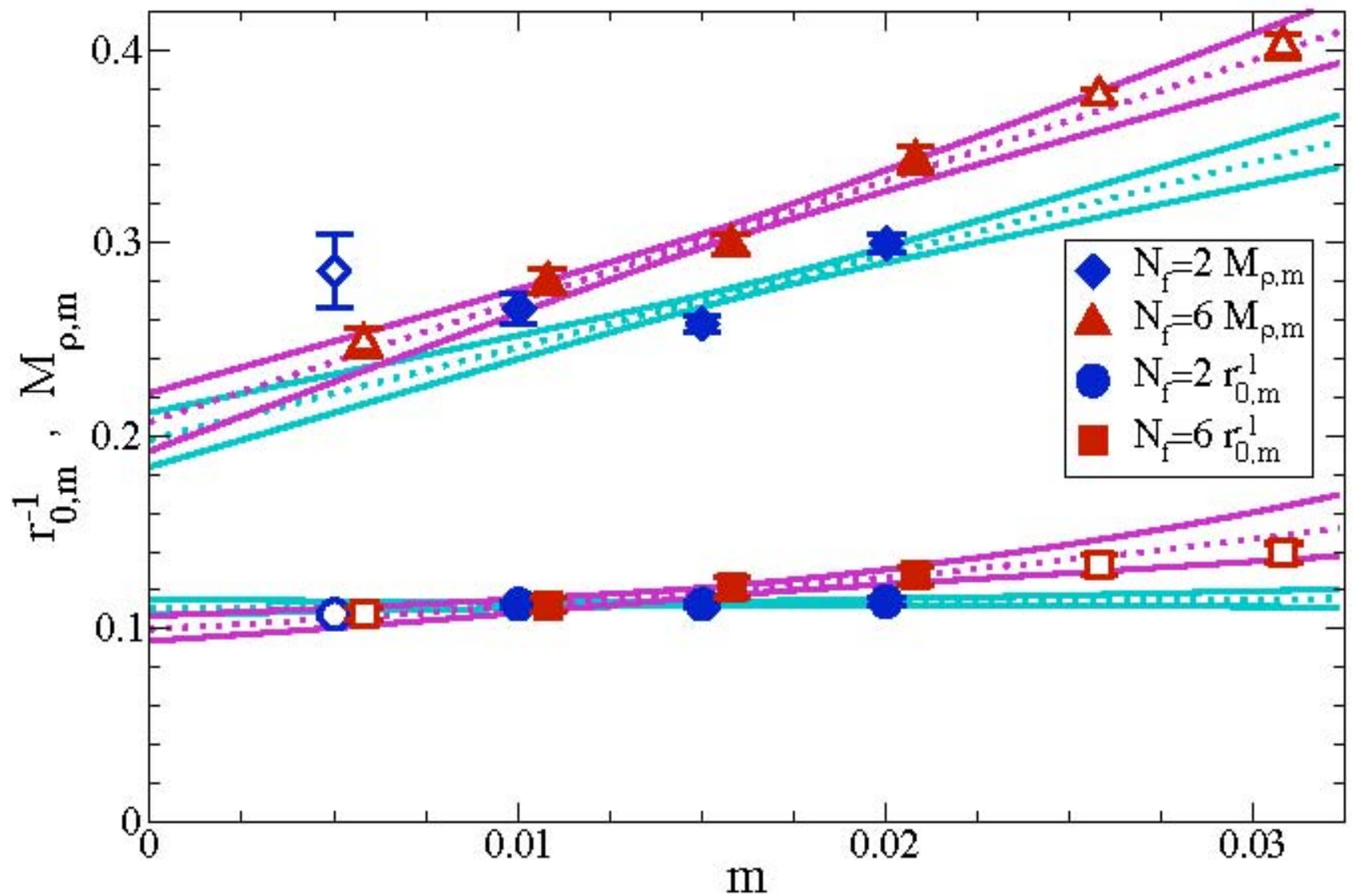
Compare the $N_f = 2$ results for $\langle \bar{\psi} \psi \rangle / 4\pi F^3$, M_ρ / F , M_{a1} / F to the QCD phenomenological values.

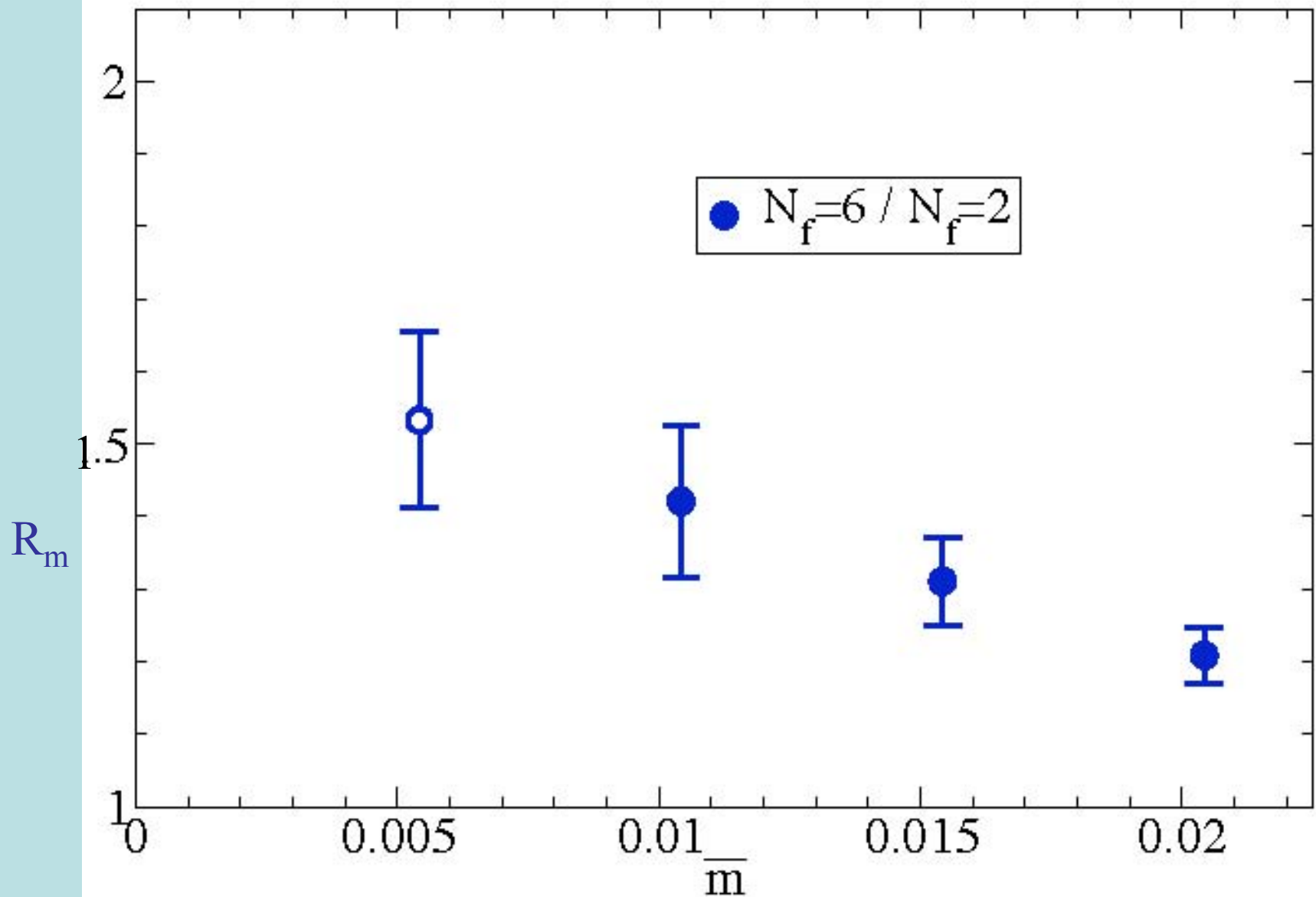
- For $N_f = 6$, compute

$\langle \bar{\psi} \psi \rangle / 4\pi F^3$ Enhancement?

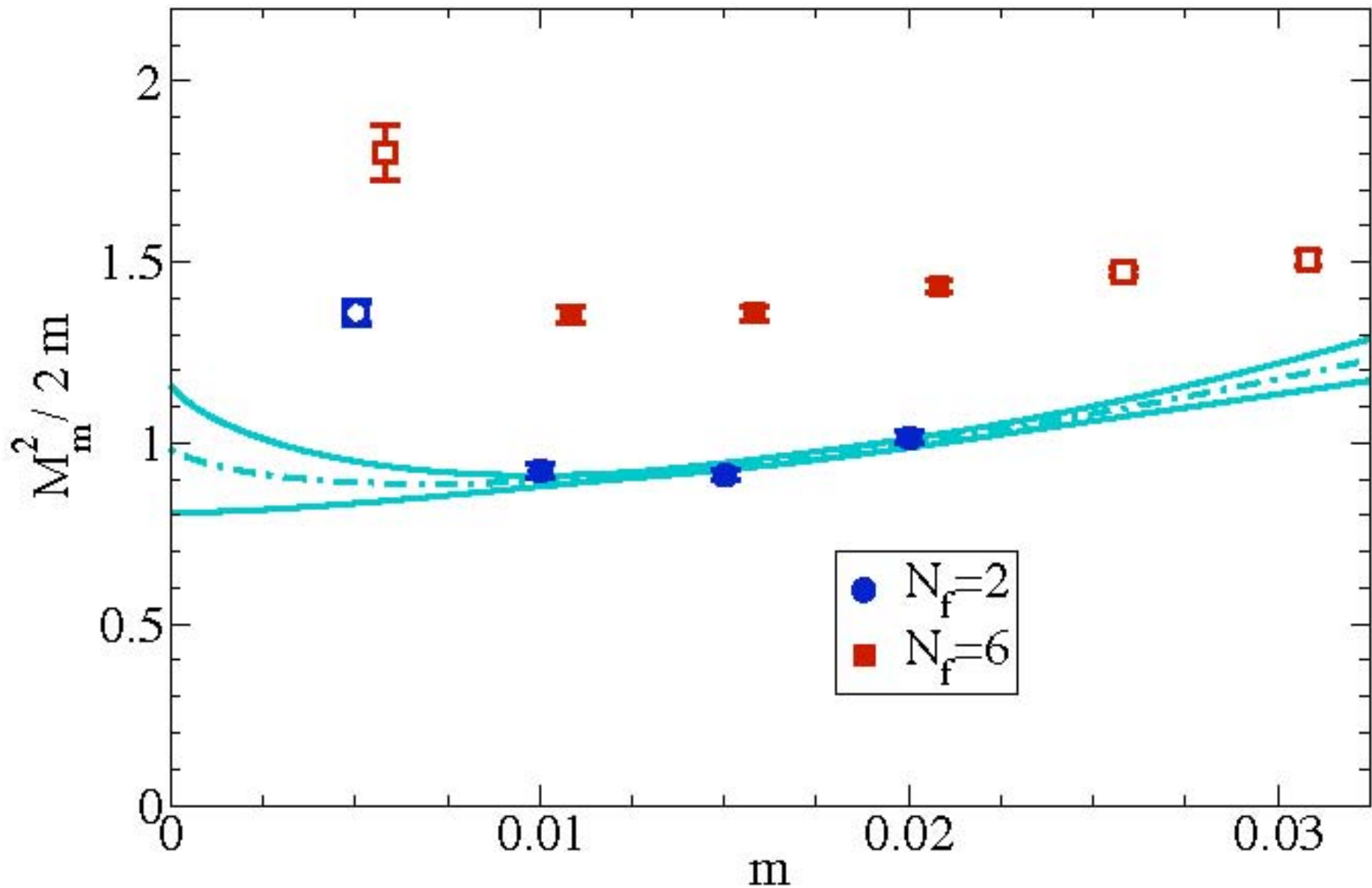
$(M_{a1} - M_\rho) / M_\rho$ \rightarrow Parity doubling?

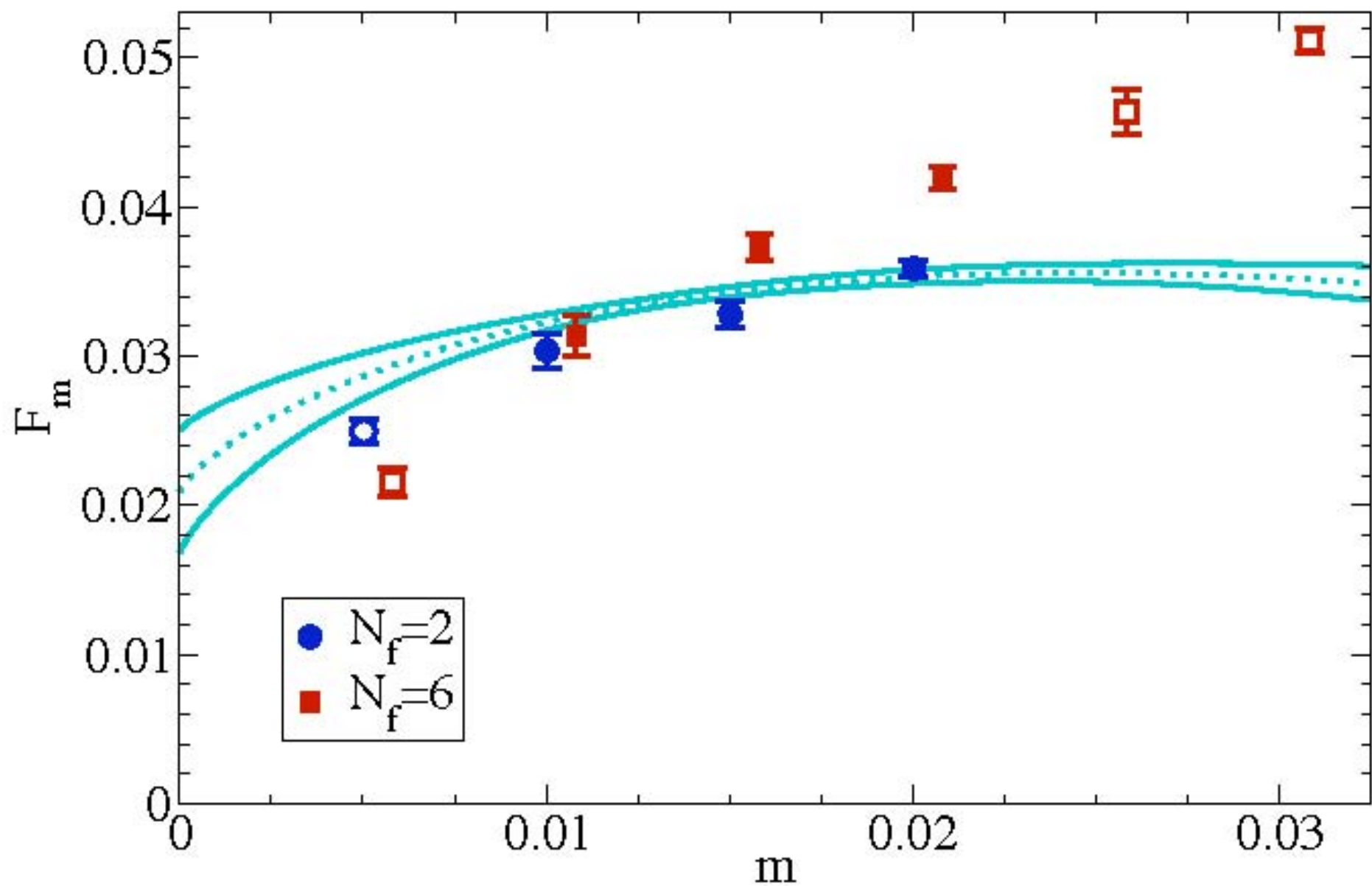
- Future: Estimate S . Then on to $N_f = 8, 10, \dots N_{fc}$ (and into the conformal window).





$$R_m = [M_m^2/2mF_m]_{6f} / [M_m^2/2mF_m]_{2f}$$





$$N_f = 2$$

- Chiral perturbation theory extrapolation:

$$\langle \bar{\psi} \psi \rangle / F^3 = 47.1 (17.6)$$

QCD Experimental Value: (renormalized to our lattice
scheme - Aoki et al hep-lat/0206013)

$$\langle \bar{\psi} \psi \rangle / F^3 = 36.2 (6.5)$$

$$N_f = 6$$

Linear Extrapolation \rightarrow

Very Conservative Lower Bound on $\langle \bar{\psi} \psi \rangle / F^2$

Very Conservative Upper Bound on F

Thus

$$\langle \bar{\psi} \psi \rangle / F^3 \geq 60.0 \text{ (8.0)}$$

Conclusion

The ratio $\langle \bar{\psi} \psi \rangle / F^3$ in an SU(3) gauge theory with N_f massless Dirac fermions in the fundamental representation, with the condensate defined by a lattice cutoff fixed in physical units, is enhanced when N_f is increased from 2 to 6 -- by at least 50% from inspection of the simulation results. Even a very conservative lower bound indicates a substantial increase. An enhancement of less than 50% would require a significant and hard-to-explain downturn in R_m . We expect the enhancement seen here, arising dominantly at the lattice scale, to be qualitatively the same as with a continuum cutoff from the onset of new physics.

Summary

1. The lattice is beginning to teach us things about theories that might describe physics beyond the standard model.
2. Work in this area is accelerating.
3. Very interesting related analytic studies
4. Lots of work to do.