Interacting Majorana Fermions and Cosmic Acceleration

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with

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A system of fermions interacting through scalar exchange, like a nucleus, exhibits negative pressure when perturbed to densities less than the equilibrium density. This preliminary study, using an adiabatic approximation, asks if there are parameter ranges that could be compatible with cosmic acceleration. We argue that the answer is affirmative.

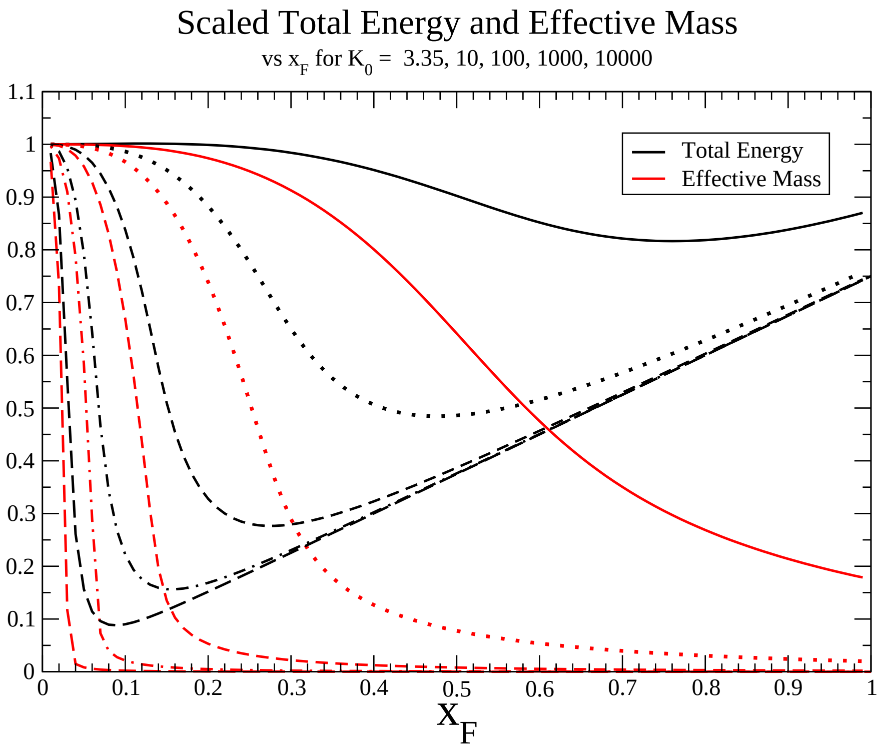
Any fermion, Majorana or Dirac, with vacuum mass m_0, interacting with coupling g with a scalar field with mass m_s, develops an effective mass

$$m^* = m_0 - \frac{g^2}{m_s^2}\langle\bar{\psi}\psi\rangle \qquad (1)$$

Using scaled variables $y = \frac{m^*}{m_0}$, $x_F = \frac{k_F}{m_0}$, $e_F = \sqrt{x_F^2 + y^2}$ and $K_0 = \frac{g^2 m_0^2}{\pi^2 m_s^2}$, this becomes

$$y = 1 - \frac{yK_0}{2}\left[e_F x_F - y^2 \ln\left(\frac{e_F + x_F}{y}\right)\right] \quad (2)$$

Define $< e >$ as the total energy per fermion, including scalar field energy.


Scaled Total Energy and Effective Mass
vs x_F for K_0 = 3.35, 10, 100, 1000, 10000
Total Energy
Effective Mass
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x_F

y and $< e >$ vs x_F

The acceleration of the time evolution of the scale parameter is given by the Friedmann Equation

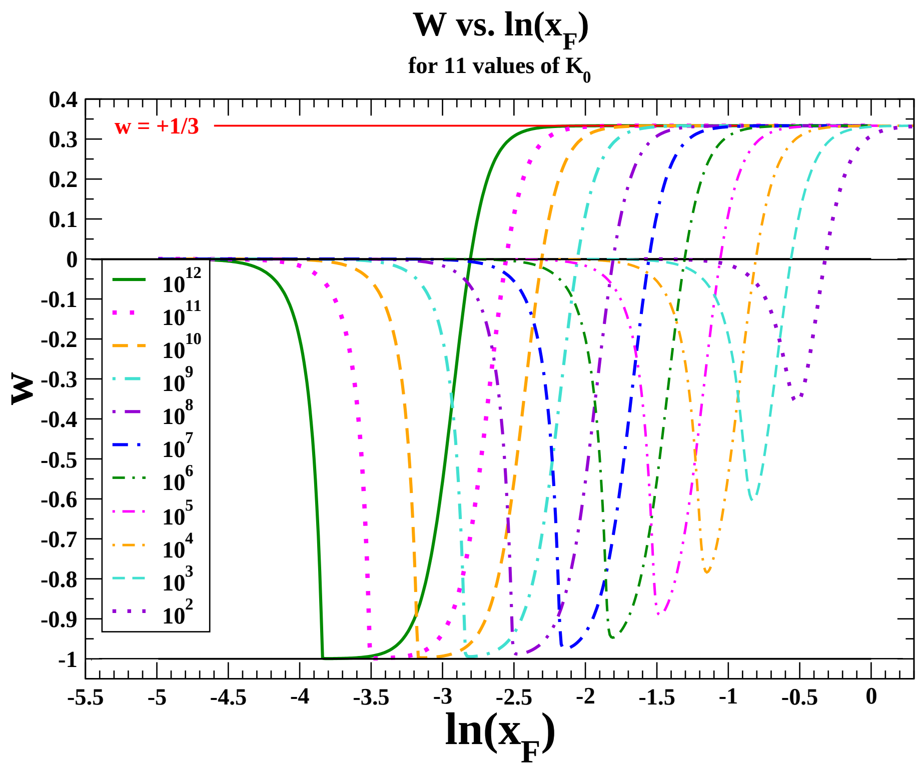
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3} \tag{3}$$

where the energy density ρ and pressure P refer to the matter being considered and Λ is the cosmological constant. The Equation of State relates ρ and P through

$$P = w\rho \tag{4}$$

For the system under consideration,

$$
\begin{aligned}
w &= \frac{1}{3}\frac{\partial < e >}{\partial x_F} \\
&= \frac{1}{3}\frac{e_F K_0 x_F^3 - 3(2-y)(1-y)}{e_F K_0 x_F^3 + (2-y)(1-y)} \qquad (5)
\end{aligned}
$$



w vs $\log(x_F)$ for 11 values of K_0

The figure suggests scaling for large enough K_0 and this can be shown with the use of analytic expansions. w becomes, to a very good approximation, $w(\xi)$ where

$$\xi = K_0 x_F^3 \tag{6}$$

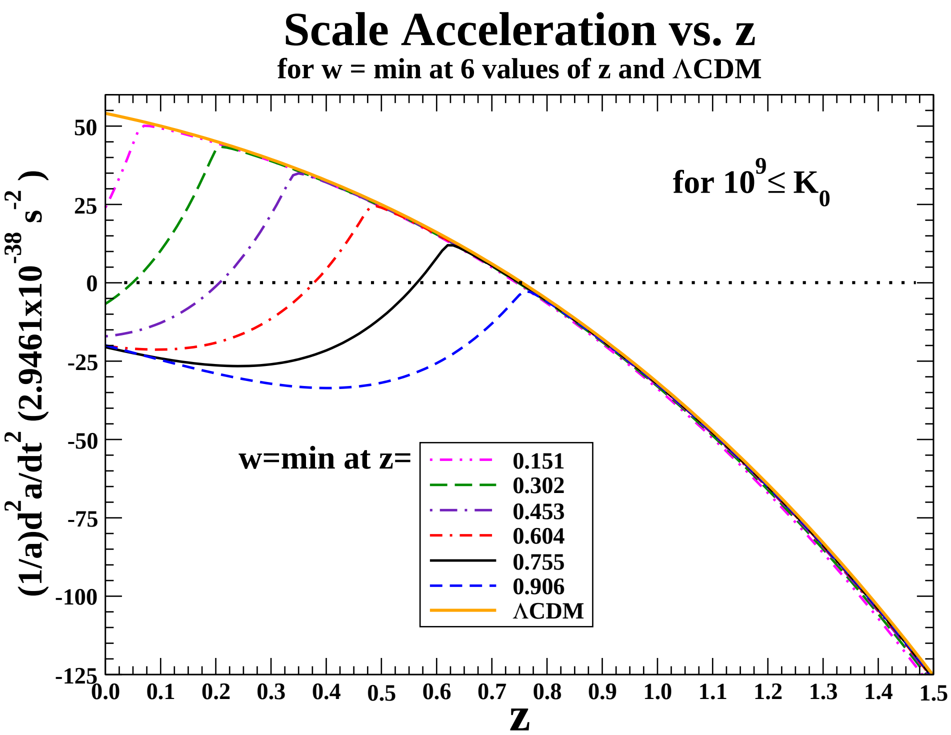
Empirically, the location of the minimum of w, x_{Fmin} is given by

$$x_{Fmin} = (3.83/K_0)^{\frac{1}{3}} \tag{7}$$

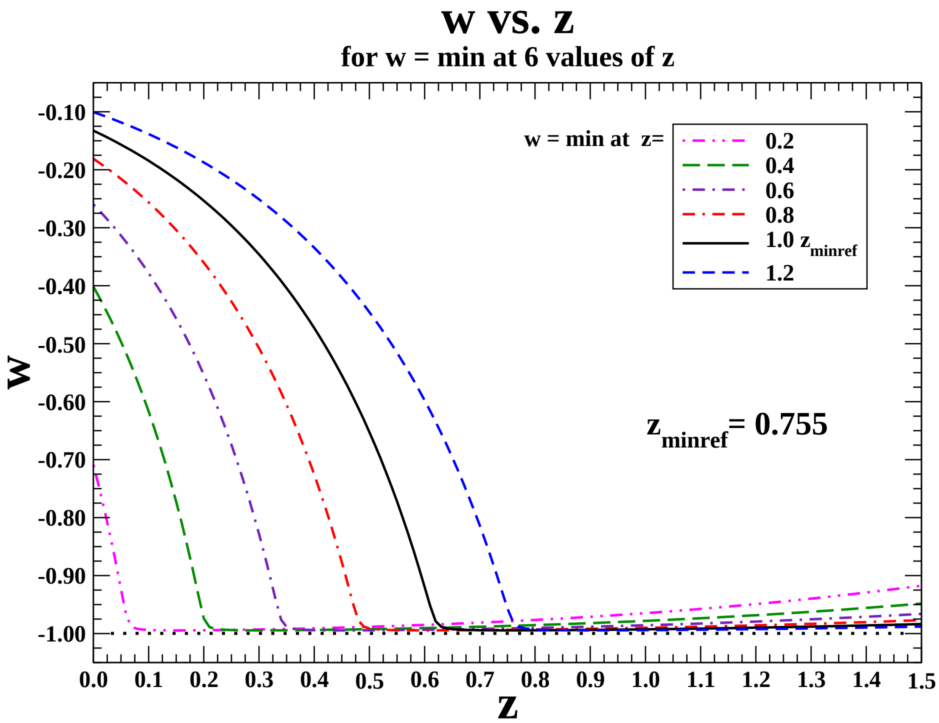
and the minimum of w is

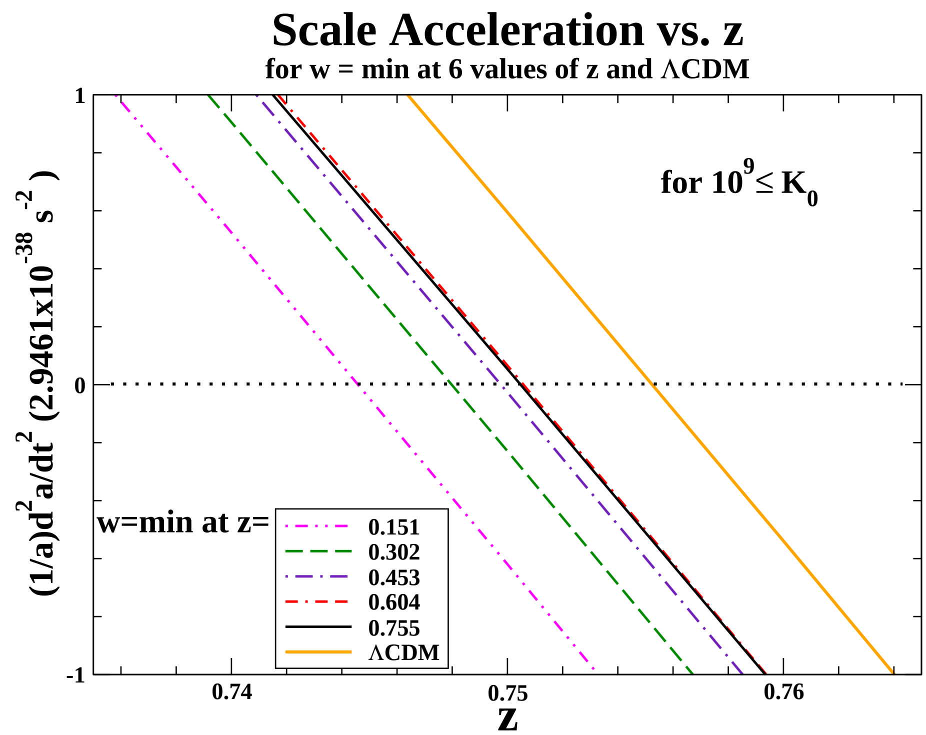
$$w \approx -1 + 2x_{Fmin} \tag{8}$$

To compare the model acceleration for a given value of K_0, including the ΛCDM estimate of the total matter density and ignoring radiation, we need to set the relation between z and x_F. We do that by choosing the value of x_F corresponding to $Z = .755$, where the acceleration goes through 0. The most obvious choice would be to use $x_{F,min}$. However, consider the example of $K_0 = 10^9$. Since w remains very close to its minimum for a range of x_F, we could choose a larger value of x_F or, equivalently, set $x_{F<min}$ to correspond to a smaller value of z.



Scale Acceleration vs. z
for w = min at 6 values of z and ΛCDM
for $10^9 \leq K_0$
(1/a)d²a/dt² (2.9461x10⁻³⁸ s⁻²)
z
w=min at z=
0.151
0.302
0.453
0.604
0.755
0.906
ΛCDM
50
25
0
-25
-50
-75
-100
-125
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5


w vs. z
for w = min at 6 values of z
w = min at z=
0.2
0.4
0.6
0.8
1.0 z_minref
1.2
z_minref= 0.755
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00
w
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
z



Scale Acceleration vs. z
for w = min at 6 values of z and ΛCDM
for 10^9 ≤ K_0
w=min at z=
0.151
0.302
0.453
0.604
0.755
ΛCDM
(1/a)d^2a/dt^2 (2.9461x10^-38 s^-2)
1
0
-1
0.74
0.75
0.76
z

To connect with reality, we need to know the density required at some value of z. Without a separate fitting of the model to data we have taken the energy density to be equal to that deduced for dark energy in the ΛCDM model, $\rho_{DE} = (2.4 meV)^4$. A good approximation in this model is

$$\rho_{DE} = \frac{m_0^4}{2\pi^2 K_0} \tag{9}$$

Look at two examples, $m_0 = 160meV, 160GeV$.

$$\begin{aligned} m_0 &= 160meV \\ K_0 &= 10^6 \\ \rho &= 33 \times 10^3 (cm)^{-3} \end{aligned} \tag{10}$$

$$\begin{aligned} m_0 &= 160GeV \\ K_0 &= 10^{54} \\ \rho &= 33 \times 10^{-9} (cm)^{-3} \end{aligned} \tag{11}$$

where the densities hold at $z \approx .75$.

$x_F \propto (1+z)$ implies that, as z becomes large for any m_0, $w \to \frac{1}{3}$ appropriate to a relativistic gas of fermions. The number density of light, active neutrinos today is about 110 per $(cm)^3$ per spin per flavor. That becomes about 185 at $z \approx .75$. Therefore BBN effectively rules out that example. Larger masses, however, will have too small a number density to affect BBN and are not constrained.

CONCLUSIONS

1) Neutral Majorana fermions, interacting with a very light scalar, could lead to $\frac{\ddot{a}}{a} > 0$ for a wide range of vacuum masses m_0 and strength parameter K_0

2) $\frac{\ddot{a}}{a}$ exhibits small differences as a function of these variables for $z > 1$. Such differences vanish as z increases and have no effect on Big Bang Nucleosynthesis.

3) Within the adiabatic approximation this system can give the same z-dependance as ΛCDM over a large range of z.

An Open Question

How well do these conclusions survive a full time dependent study? In particular, the long run in z of $w \approx -1$ implies a long period when the system is smoothly departing from its equilibrium density. This is not stable. If the system breaks up into smaller systems with densities near the equilibrium density the effect in the Friedmann Equation will be to add more matter and the apparent tracking with ΛCDM will be destroyed.