Planck Scale Cosmology in Resummed Quantum Gravity

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Outline:

- **Introduction**
- **Review of Feynman's Formulation of Einstein's Theory**
- **Resummed Quantum Gravity**
- **Recent Applications: Asymptotic Safety &**

  Planck Scale Cosmology
- **Conclusions**

Papers by B.F.L. Ward, S. Jadach *et al.*: MPL**A23**(2008)3299; IJMPD**17**(2008)627;JCAP**0402**(2004)011;MPL**A19**(2004)143;**A17** (2002) 2371, CPC**102** (1997) 229, CPC**124** (2000) 233, CPC**130** (2000) 260, CPC**140** (2001) 432, CPC**140** (2001) 475, and references therein.

Motivation

- NEWTON'S LAW: MOST BASIC ONE IN PHYSICS – TAUGHT TO ALL BEGINNING STUDENTS

- ALBERT EINSTEIN: SPECIAL CASE OF THE SOLUTIONS OF THE CLASSICAL FIELD EQUATIONS OF THE GENERAL THEORY OF RELATIVITY

  $g_{00} = 1 + 2\varphi \Rightarrow \nabla^2\varphi = 4\pi G_N \rho$

  from

  $R^{\alpha\gamma} - \frac{1}{2}g^{\alpha\gamma}R + \Lambda g^{\alpha\gamma} = -8\pi G_N T^{\alpha\gamma}$, etc.,

- HEISENBERG & SCHROEDINGER, FOLLOWING BOHR: QUANTUM MECHANICS

  $\Leftrightarrow$ EVEN WITH TREMENDOUS PROGRESS: QUANTUM FIELD THEORY, SUPERSTRINGS, LOOP QUANTUM GRAVITY, ETC.,

  NO SATISFACTORY TREATMENT OF THE QUANTUM MECHANICS OF NEWTON'S LAW IS KNOWN TO BE PHENOMENOLOGICALLY CORRECT

TODAY'S TALK

- WE APPLY A NEW APPROACH(MPL**A17** (2002) 2371;**A19** (2004) 143; JCAP **0402** (2004) 011; IJMPD **17** (2008) 627), BUILDING ON PREVIOUS WORK:
  R.P. FEYNMAN: Acta Pys. Pol. **24** (1963) 697; *FEYNMAN LECTURES ON GRAVITATION*, eds. F.B. Moringo and W.G. Wagner, (Caltech, Pasadena, 1971).

- BASIC IDEA: QUANTUM GRAVITY IS A POINT PARTICLE QUANTUM FIELD THEORY AND ITS APPARENT BAD UV BEHAVIOR IS DUE TO OUR NAIVETE –
  NOTHING FUNDAMENTAL PREVENTS
  THE UNION OF BOHR AND EINSTEIN.

- WEINBERG, IN *GENERAL RELATIVITY*, eds. S.W. Hawking and W. Israel,(Cambridge Univ. Press, Cambridge, 1979) p.790
  FOUR APPROACHES TO UV BEHAVIOR OF QUANTUM GRAVITY (QG)
  - Extended Theories Of Gravitation: Supersymmetric Theories - Superstrings;Loop Quantum Gravity
  - Resummation⇐ TODAY'S TALK – NEW VERSION:
    - ∗ In non-Abelian gauge theories, the Källén-Lehmann representation cannot be used to show that $Z_3(g)$ is formally less than 1 ⇒ Weinberg's argument that $\rho_{K-L}(\mu) \geq 0$ prevents graviton propagator from falling faster than $1/k^2$ does not hold in such theories, as he has intimated himself.
  - Composite Gravitons
  - Asymptotic Safety: Fixed Point Theory (See Lauscher & Reuter, PRD**66**(2002)025026; Bonanno & Reuter, PRD**62**(2000)043008);**65**(2002) 043508; arXiv:0803.2546, and references therein.–SUBJECT OF THIS CONFERENCE

NEW APPROACH: RESUMMED QUANTUM GRAVITY

IN Mod. Phys. Lett.**A17**(2002)2371, WE SHOWED THAT RESUMMATION CURES THE BAD UV BEHAVIOR OF EINSTEIN'S THEORY

$\Leftrightarrow$

MANY CONSEQUENCES AND TESTS

- A SM MASSIVE POINT PARTICLE OF MASS m HAS A NONZERO SCHWARZSCHILD RADIUS $r_S = 2(m/M_{Pl}^2)$ AND BY EINSTEIN'S THEORY IS CLASSICALLY A BLACK HOLE– WE CAN ONLY SEE ITS HAWKING RADIATION – WE SHOW IN IJMP**17** (2008) 627 THAT RQG OBVIATES THIS.
- TODAY, WE ADDRESS SUCH ISSUES AS WELL AS PLANCK-SCALE COSMOLOGY IN OUR NEW APPROACH TO QG FROM PERSPECTIVE OF ASYMPTOTIC SAFETY.

Review of Feynman's Formulation of Einstein's Theory

For the known world, we have the generally covariant Lagrangian

$$\mathcal{L}(x) = \frac{1}{2\kappa^2}\sqrt{-g}R + \sqrt{-g}L^{\mathcal{G}}_{SM}(x) \qquad (1)$$

- R is the curvature scalar, and we for now set the small observed cosmological constant Λ to zero
- $-g = -detg_{\mu\nu}$
- $\kappa = \sqrt{8\pi G_N} \equiv \sqrt{8\pi/M^2_{Pl}}$, where G_N is Newton's constant,
- SM Lagrangian density $= L^{\mathcal{G}}_{SM}(x)$

One gets $L^{\mathcal{G}}_{SM}(x)$ from the usual SM Lagrangian density as follows:

- Note that $\partial_\mu\phi(x)$ is already generally covariant for any scalar field ϕ.
- Note that the only derivatives of the vector fields in the SM Lagrangian density occur in their curls, $\partial_\mu A^J_\nu(x) - \partial_\nu A^J_\mu(x)$, which are also already generally covariant.

- Thus, we only need to give a rule for the fermionic terms. $\Rightarrow$

  We introduce a differentiable structure with $\{\xi^a(x)\}$ as locally inertial coordinates and an attendant vierbein field $e^a_\mu \equiv \partial\xi^a/\partial x^\mu$ with indices that carry the vector representation for the flat locally inertial space, a, and for the manifold of space-time, μ, with the identification of the space-time base manifold metric as $g_{\mu\nu} = e^a_\mu e_{a\nu}$ where the flat locally inertial space indices are to be raised and lowered with Minkowski's metric η_{ab} as usual.

  Associating the usual Dirac gamma matrices $\{\gamma_a\}$ with the flat locally inertial space at x, we define base manifold Dirac gamma matrices by $\Gamma_\mu(x) = e^a_\mu(x)\gamma_a$.

Then the spin connection,

$$\omega_{\mu b}^{a} = -\frac{1}{2}e^{a\nu}\left(\partial_\mu e_\nu^b - \partial_\nu e_\mu^b\right) + \frac{1}{2}e^{b\nu}\left(\partial_\mu e_\nu^a - \partial_\nu e_\mu^b\right)$$
$$+\frac{1}{2}e^{a\rho}e^{b\sigma}\left(\partial_\rho e_{c\sigma} - \partial_\sigma e_{c\rho}\right)e_\mu^c$$

when there is no torsion, allows us to identify the generally covariant Dirac operator for the SM fields by the substitution $i\not\partial \rightarrow i\Gamma(x)^\mu\left(\partial_\mu + \frac{1}{2}\omega_{\mu b}{}^a\Sigma^b{}_a\right)$, where we have $\Sigma^b{}_a = \frac{1}{4}\left[\gamma^b, \gamma_a\right]$ everywhere in the SM Lagrangian density. This will generate $L_{SM}^{\mathcal{G}}(x)$ from the usual SM Lagrangian density $L_{SM}(x)$ as it is given in the papers of Hollik, Bardin, Passarino, etc., for example.

SM $\Leftrightarrow$ Many Massive Point Particles.

To begin the study of their quantum gravity interactions, we follow Feynman and treat spin as an inessential complication. We come back to a spin-dependent analysis presently.

We replace $L^{\mathcal{G}}_{SM}(x)$ in (1) with the simplest case for our studies, that of a free scalar field, a free physical Higgs field, $\varphi(x)$, with a rest mass believed to be less than 400 GeV and known to be greater than 114.4 GeV with a 95% CL. We are then led to consider the representative model $\{$**R.P. Feynman, Acta Phys. Pol. 24 (1963) 697;** *Feynman Lectures on Gravitation*, **eds. F.B. Moringo and W.G. Wagner, (Caltech, Pasadena, 1971).** $\}$

$$
\begin{aligned}
\mathcal{L}(x) &= -\frac{1}{2\kappa^2} R\sqrt{-g} + \frac{1}{2}\left(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - m_o^2\varphi^2\right)\sqrt{-g} \\
&= \quad \frac{1}{2}\left\{h^{\mu\nu,\lambda}\bar{h}_{\mu\nu,\lambda} - 2\eta^{\mu\mu'}\eta^{\lambda\lambda'}\bar{h}_{\mu_\lambda,\lambda'}\eta^{\sigma\sigma'}\bar{h}_{\mu'\sigma,\sigma'}\right\} \\
&\quad + \frac{1}{2}\left\{\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\right\} - \kappa h^{\mu\nu}\left[\overline{\varphi_{,\mu}\varphi_{,\nu}} + \frac{1}{2}m_o^2\varphi^2\eta_{\mu\nu}\right] \\
&\quad - \kappa^2\left[\frac{1}{2}h_{\lambda\rho}\bar{h}^{\rho\lambda}\left(\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\right) - 2\eta_{\rho\rho'}h^{\mu\rho}\bar{h}^{\rho'\nu}\varphi_{,\mu}\varphi_{,\nu}\right] + \cdots
\end{aligned}
\tag{2}
$$

where $\varphi_{,\mu} \equiv \partial_\mu \varphi$ and we have

- $g_{\mu\nu}(x) = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)$,
  $\eta_{\mu\nu} = diag\{1, -1, -1, -1\}$
- $\bar{y}_{\mu\nu} \equiv \frac{1}{2}\left(y_{\mu\nu} + y_{\nu\mu} - \eta_{\mu\nu} y_\rho{}^\rho\right)$ for any tensor $y_{\mu\nu}$
- Feynman rules already worked-out by Feynman (*op. cit.*), where we use his gauge, $\partial^\mu \bar{h}_{\nu\mu} = 0$

$\Leftrightarrow$ Quantum Gravity is just another quantum field theory where the metric now has quantum fluctuations as well.

For example, the one-loop corrections to the graviton propagator due to matter loops is just given by the diagrams in Fig. 1.

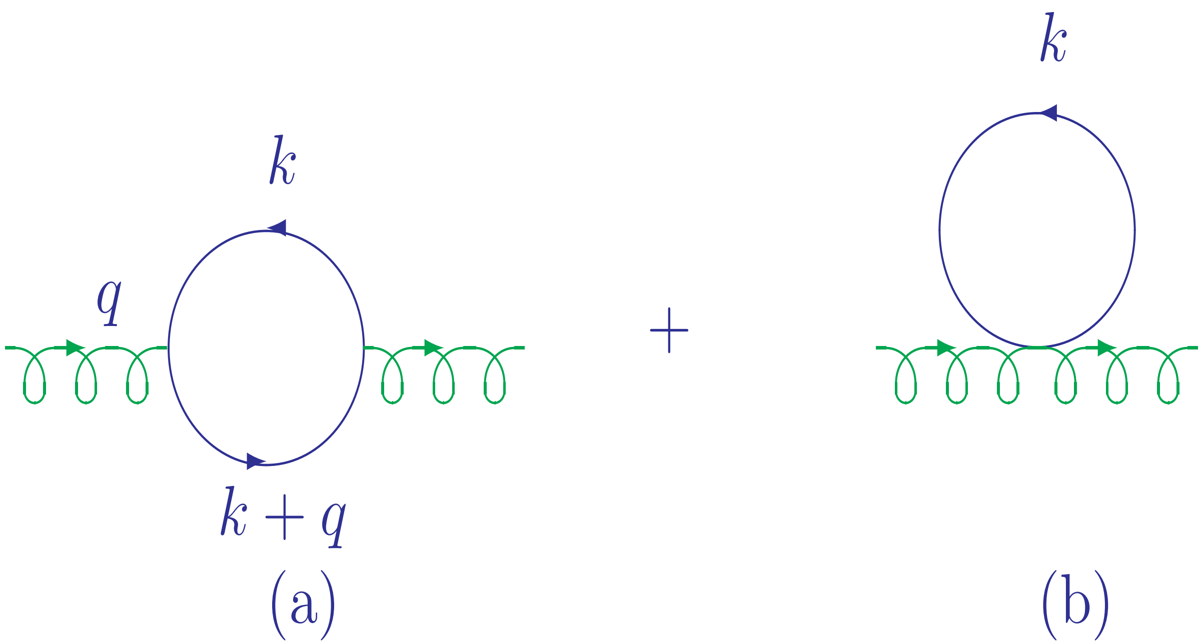

Figure 1: The scalar one-loop contribution to the graviton propagator. q is the 4-momentum of the graviton.

These graphs already illustrate the QG's BAD UV behavior.

UV DIVERGENCE DEGREES

- **(a) AND (b) HAVE SUPERFICIAL** $D = +4$
- **AFTER TAKING GAUGE INVARIANCE INTO ACCOUNT, WE STILL EXPECT** $D_{eff} \geq 0$
- **HIGHER LOOPS GIVE HIGHER VALUES OF** D_{eff}
- **CONCLUSION: QG IS NONRENORMALIZABLE**

WE SHOW THESE ESTIMATES EXPLICITLY SHORTLY.

PHYSICAL EFFECT

DEEP UV EUCLIDEAN REGIME OF FEYNMAN INTEGRAL

- GRAVITATIONAL FORCE IS ATTRACTIVE AND $\propto$ TO MASS^2
- DEEP UV EUCLIDEAN REGIME $\Leftrightarrow$ LARGE NEGATIVE MASS^2
- IN THIS REGIME, GRAVITY IS REPULSIVE
- PROPAGATION BETWEEN TWO DEEP EUCLIDEAN POINTS SEVERELY SUPPRESSED IN EXACT SOLUTIONS OF THE THEORY $\Rightarrow$
- RESUM LARGE SOFT GRAVITON EFFECTS TO GET MORE PHYSICALLY CORRECT RESULTS, A DYNAMICAL REALIZATION OF ASYMPTOTIC SAFETY

RESUMMED QUANTUM GRAVITY

WE WILL YFS RESUM THE PROPAGATORS IN THE THEORY:

⇒ FROM THE EW RESUMMED FORMULA OF YFS

$$\Sigma_F(p) = e^{\alpha B''_\gamma}\left[\Sigma'{}_F(p) - S_F^{-1}(p)\right] + S_F^{-1}(p), \tag{3}$$

WHICH ⇒

$$iS'_F(p) = \frac{ie^{-\alpha B''_\gamma}}{S_F^{-1}(p) - \Sigma'{}_F(p)}, \tag{4}$$

FOR

$$\Sigma'{}_F(p) = \sum_{n=1}^{\infty} \Sigma'{}_{Fn}, \tag{5}$$

WE NEED TO FIND FOR QUANTUM GRAVITY THE ANALOGUE OF

$$\alpha B''_\gamma = \int d^4\ell \frac{S''(k,k,\ell)}{\ell^2 - \lambda^2 + i\epsilon} \tag{6}$$

WHERE $\lambda \equiv$ IR CUT-OFF AND

$$S''(k,k,\ell) = \frac{-i8\alpha}{(2\pi)^3} \frac{kk'}{(\ell^2 - 2\ell k + \Delta + i\epsilon)(\ell^2 - 2\ell k' + \Delta' + i\epsilon)}\Big|_{k=k'}, \quad (7)$$

$\Delta = k^2 - m^2$, $\Delta' = {k'}^2 - m^2$.

TO THIS END, NOTE ALSO

$$\alpha B''_\gamma = \int \frac{d^4\ell}{(2\pi)^4} \frac{-i\eta^{\mu\nu}}{(\ell^2 - \lambda^2 + i\epsilon)} \frac{-ie(2ik_\mu)}{(\ell^2 - 2\ell k + \Delta + i\epsilon)} \frac{-ie(2ik'_\nu)}{(\ell^2 - 2\ell k' + \Delta' + i\epsilon)}\Big|_{k=k'}. \quad (8)$$

$\Rightarrow$ WE FOLLOW WEINBERG/THE FEYNMAN RULES

AND IDENTIFY THE CONSERVED GRAVITON CHARGES AS

$e \longrightarrow \kappa k_\rho$ FOR SOFT EMISSION FROM k

$\Rightarrow$ WE GET THE ANALOGUE , $-B''_g(k)$, OF $\alpha B''_\gamma$ BY

- REPLACING THE γ PROPAGATOR IN (8) BY THE GRAVITON PROPAGATOR,

$$\frac{i\frac{1}{2}(\eta^{\mu\nu}\eta^{\bar\mu\bar\nu} + \eta^{\mu\bar\nu}\eta^{\bar\mu\nu} - \eta^{\mu\bar\mu}\eta^{\nu\bar\nu})}{\ell^2 - \lambda^2 + i\epsilon}$$

,

- BY REPLACING THE QED CHARGES BY THE CORRESPONDING GRAVITY CHARGES $\kappa k_{\bar{\mu}},\ \kappa k'_{\bar{\nu}}$

$\Rightarrow$

$$B''_g(k) = -2i\kappa^2 k^4 \frac{\int d^4\ell}{16\pi^4} \frac{1}{\ell^2 - \lambda^2 + i\epsilon} \frac{1}{(\ell^2 + 2\ell k + \Delta + i\epsilon)^2} \quad (9)$$

AND

$$i\Delta'_F(k)|_{Resummed} = \frac{ie^{B''_g(k)}}{(k^2 - m^2 - \Sigma'_s + i\epsilon)} \quad (10)$$

THIS IS THE BASIC RESULT. THE EXACTNESS OF THIS RESULT FOR ALL k IS PROVED IN OUR PAPERS AND BY YFS.

NOTE THE FOLLOWING:

- Σ'_s STARTS IN $\mathcal{O}(\kappa^2)$, SO WE MAY DROP IT IN CALCULATING ONE-LOOP EFFECTS.

- EXPLICIT EVALUATION GIVES, FOR THE DEEP UV REGIME,

$$B''_g(k) = \frac{\kappa^2|k^2|}{8\pi^2} \ln\left(\frac{m^2}{m^2 + |k^2|}\right), \quad (11)$$

$\Rightarrow$ **THE RESUMMED PROPAGATOR FALLS FASTER THAN ANY POWER OF $|k^2|$!**

- **NOTE: IN EUCLIDEAN REGIME, $-|k^2| = k^2$ SO THERE IS TRIVIALLY NO ANALYTICITY ISSUE HERE.**

• **IF m VANISHES, USING THE USUAL $-\mu^2$ NORMALIZATION POINT WE GET $B''_g(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{\mu^2}{|k^2|}\right)$ WHICH AGAIN VANISHES FASTER THAN ANY POWER OF $|k^2|$!**

THIS MEANS THAT ONE-LOOP CORRECTIONS ARE FINITE!

INDEED, ALL QUANTUM GRAVITY LOOPS ARE UV FINITE!

ALL ORDERS PROOF

Consider the entire theory from (2) to all orders in κ:

$$\mathcal{L}(x) = \mathcal{L}_0(x) + \sum_{n=1}^{\infty} \kappa^n \mathcal{L}_I^{(n)}(x) \tag{12}$$

– the interactions, including the ghost interactions, are the terms of $\mathcal{O}(\kappa^n), n \geq 1$.

$$\mathcal{L}_I^{(n)}(x) = \sum_{\ell=1}^{m_n} \mathcal{L}_{I,\ell}^{(n)}(x). \tag{13}$$

$\mathcal{L}_{I,\ell}^{(n)}$ **has dimension $d_{n,\ell}$.**

Let $d_n^M = \max_\ell\{d_{n,\ell}\}$. $\Rightarrow$ The maximum power of momentum at any vertex in $\mathcal{L}_I^{(n)}$ is $\bar{d}_n^M = \min\{d_n^M - 3, 2\}$ and is finite.

Note, in any gauge,

$$i\mathcal{D}^{(0)}_{F\alpha_1\cdots;\alpha'_1\cdots}(k)|_{YFS-resummed} = \frac{iP_{\alpha_1\cdots;\alpha'_1\cdots}e^{B''_g(k)}}{(k^2-m^2+i\epsilon)}, \tag{14}$$

so that it is also exponentially damped at high energy in the deep Euclidean regime (DER).

Now consider any 1PI vertex Γ_N with $[N] \equiv n_1 + n_2$ amputated external legs, where $N = (n_1, n_2)$, when $n_1(n_2)$ is the respective number of graviton(scalar) external lines.

At its zero-loop order, there are only tree contributions which are manifestly UV finite.

Consider the first loop ($\mathcal{O}(\kappa^2)$) corrections to Γ_N. There must be at least one improved exponentially damped propagator in the respective loop contribution and at most two vertices so that the maximum power of momentum in the numerator of the loop due to the vertices is $\max\{2\bar{d}_1^M, \bar{d}_2^M\}$ and is finite.

The exponentially damped propagator $\Rightarrow$ the loop integrals finite $\Rightarrow$ the entire one-loop ($\mathcal{O}(\kappa^2)$) contribution is finite.

Corollary: If Γ_N vanishes in tree approximation, we can conclude that its first non-trivial contributions at one-loop are all finite, due to the exponentially damped propagator. Using induction on the number of loops, it is possible to prove finiteness of all loop contributions– See MPLA17(2002)2371.

Pictorially, we illustrate the type of situations we have in Fig. 2.

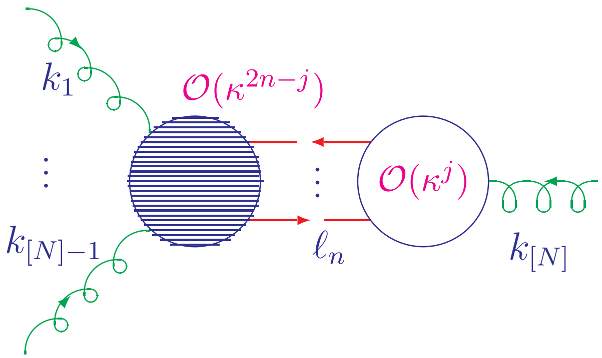

Fig.2. The typical contribution we encounter in Γ_N at the n-loop level; ℓ_n is the n-th loop momentum and is precisely the momentum of the indicated YFS-resummed improved Born propagator.

- **Consistent with asymptotic safety approach: Reuter et al., Percacci et al., Litim, etc.**
- **Consistent with recent Hopf-algebraic Dyson-Schwinger Eqn. renormalization theory results of Kreimer - Ann.Phys.321(2006)2757; 323(2008)49.**

Resumming the Vertex Corrections

- In the IR this does not change our damping in the deep UV: The IR virtual function which exponentiates in conjunction with $B_{g''}$ above is given by

$$B_{IR}(p',p) = \frac{-i\int d^4k}{32\pi^4}\frac{\frac{1}{2}(\eta^{\mu\nu}\eta^{\bar{\mu}\bar{\nu}}+\eta^{\mu\bar{\nu}}\eta^{\bar{\mu}\nu}-\eta^{\mu\bar{\mu}}\eta^{\nu\bar{\nu}})}{k^2-m_g^2+i\epsilon}$$
$$\left[\frac{V(p-k,p)_{\mu\bar{\mu}}}{k^2-2kp+\Delta(p)+i\epsilon}-\frac{V(p',p'-k)_{\mu\bar{\mu}}}{k^2-2kp'+\Delta(p')+i\epsilon}\right]$$
$$\left[\frac{V(p-k,p)_{\nu\bar{\nu}}}{k^2-2kp+\Delta(p)+i\epsilon}-\frac{V(p',p'-k)_{\nu\bar{\nu}}}{k^2-2kp'+\Delta(p')+i\epsilon}\right]$$
$$+\cdots$$

with $\Delta(p)=p^2-m^2$ and with the IR limit form

$$V(p-k,p)_{\mu\bar{\mu}} = -\kappa((p-k)_\mu p_{\bar{\mu}}+(p-k)_{\bar{\mu}}p_\mu). \tag{15}$$

$\Rightarrow$ **Vertex resummation does not change our deep UV behavior.**

Explicit Finiteness of $\Sigma'^{(1)}$

$$\Sigma_s'^{(1)}(k) = \Sigma_s^{(1)}(k) - B_g''(k)\Delta_F^{-1}(k) \tag{16}$$

$\Rightarrow$

$$\begin{aligned}
\Sigma_s'^{(1)}(k) = -\kappa^2 \frac{\int d^4\ell}{(2\pi)^4}\Bigg\{ \Big[& (2k^\mu k^\nu)\mathcal{P}_{\mu\nu;\mu'\nu'}(\ell)(2k^{\mu'}k^{\nu'})\frac{\ell^2+2\ell k+2(k^2-m^2)}{\ell^2+2\ell k+k^2-m^2+i\epsilon} \\
&+ \Delta V^{\mu\nu}(k,\ell)\mathcal{P}_{\mu\nu;\mu'\nu'}(\ell)(2k^{\mu'}k^{\nu'}) + (2k^\mu k^\nu)\mathcal{P}_{\mu\nu;\mu'\nu'}(\ell)\Delta V^{\mu'\nu'}(k,\ell) \\
&+ \Delta V^{\mu\nu}(k,\ell)\mathcal{P}_{\mu\nu;\mu'\nu'}(\ell)\Delta V^{\mu'\nu'}(k,\ell)\Big] \frac{e^{\frac{\kappa^2|(k+\ell)^2|}{8\pi^2}\ln\left(\frac{m^2}{m^2+|(k+\ell)^2|}\right)}}{(k+\ell)^2-m^2+i\epsilon}\frac{e^{\frac{\kappa^2|\ell^2|}{8\pi^2}\ln\left(\frac{\mu^2}{|\ell^2|}\right)}}{\ell^2+i\epsilon} \\
&+ \Big[\frac{1}{2}(k^2-m^2)\left(\mathcal{P}_{\lambda\rho;}{}^{\rho\lambda}(\ell) + \mathcal{P}_{\lambda\rho;}{}^{\lambda\rho}(\ell) - \mathcal{P}_\lambda{}^\lambda{}_{;\lambda'}{}^{\lambda'}(\ell)\right) \\
&- (2k^\mu k^\nu)\left(\mathcal{P}_{\mu\rho;}{}^\rho{}_\nu(\ell) + \mathcal{P}_{\mu\rho;\nu}{}^\rho(\ell) - \mathcal{P}_{\mu\nu;\rho}{}^\rho(\ell)\right)\Big] \frac{e^{\frac{\kappa^2|\ell^2|}{8\pi^2}\ln\left(\frac{\mu^2}{|\ell^2|}\right)}}{\ell^2+i\epsilon}\Bigg\},
\end{aligned} \tag{17}$$

where $\Delta V^{\mu\nu}(k,\ell) = k^\mu \ell^\nu + k^\nu \ell^\mu - (k^2 - m^2 + k\ell)\eta^{\mu\nu}$. $\Rightarrow$ **UV FINITE!**

RECENT APPLICATIONS: ASYMPT SFTY,PLANCK SCALE COS.

CONSIDER THE GRAVITON PROPAGATOR IN THE THEORY OF GRAVITY COUPLED TO A MASSIVE SCALAR(HIGGS) FIELD(Feynman). WE HAVE THE GRAPHS IN Fig. 2 IN ADDITIOn TO THAT IN Fig. 1.

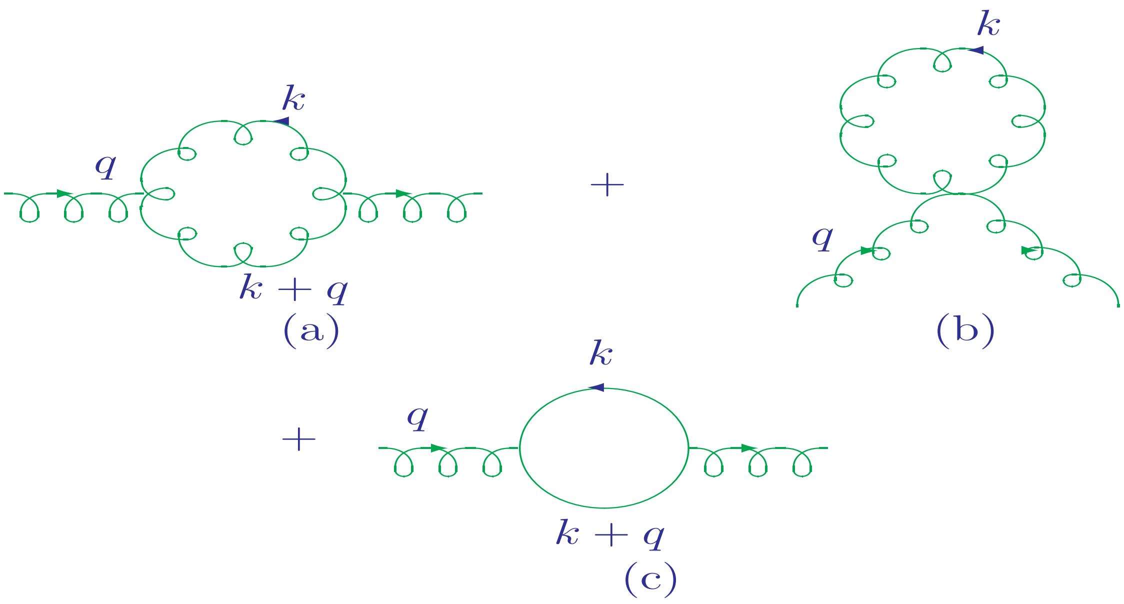

Figure 2: The graviton((a),(b)) and its ghost((c)) one-loop contributions to the graviton propagator. q **is the 4-momentum of the graviton.**

USING THE RESUMMED THEORY, WE GET THAT THE NEWTON POTENTIAL BECOMES

$$\Phi_N(r) = -\frac{G_N M}{r}(1 - e^{-ar}), \tag{18}$$

FOR

$$a \cong 0.210 M_{Pl}. \tag{19}$$

CONTACT WITH AYMPTOTIC SAFETY APPROACH

- OUR RESULTS IMPLY

$$G(k) = G_N/(1 + \frac{k^2}{a^2})$$

$\Rightarrow$ FIXED POINT BEHAVIOR FOR

$k^2 \rightarrow \infty$,

IN AGREEMENT WITH THE PHENOMENOLOGICAL ASYMPTOTIC SAFETY APPROACH OF BONNANNO & REUTER IN PRD62(2000) 043008.

- OUR RESULTS IMPLY THAT AN ELEMENTARY PARTICLE HAS

  NO HORIZON WHICH ALSO AGREES WITH BONNANNO'S & REUTER'S

  RESULT THAT A BLACK HOLE WITH A MASS LESS THAN

  $M_{cr} \sim M_{Pl}$

  HAS NO HORIZON.

  BASIC PHYSICS:

  $G(k)$ VANISHES FOR $k^2 \rightarrow \infty$.

- A FURTHER "AGREEMENT": FINAL STATE OF HAWKING RADIATION OF AN ORIGINALLY VERY MASSIVE BLACKHOLE

  BECAUSE OUR VALUE OF THE COEFFICIENT,

  $$\frac{1}{a^2},$$

  OF k^2 IN THE DENOMINATOR OF $G(k)$

  AGREES WITH THAT FOUND BY BONNANNO & REUTER(B-R),

  IF WE USE THEIR PRESCRIPTION FOR THE

  RELATIONSHIP BETWEEN k AND r

  IN THE REGIME WHERE THE LAPSE FUNCTION VANISHES,

  WE GET THE SAME HAWKING RADIATION PHENOMEMNOLOGY AS THEY DO:

  THE BLACK HOLE EVAPORATES IN THE B-R ANALYSIS UNTIL IT REACHES A MASS

  $$M_{cr} \sim M_{Pl}$$

  AT WHICH THE BEKENSTEIN-HAWKING TEMPERATURE VANISHES,

  LEAVING A PLANCK SCALE REMNANT.

- FATE OF REMNANT? IN hep-ph/0503189 $\Rightarrow$ OUR QUANTUM LOOP EFFECTS COMBINED WITH THE $G(r)$ OF B-R IMPLY HORIZON OF THE PLANCK SCALE REMNANT IS OBVIATED – CONSISTENT WITH RECENT RESULTS OF HAWKING.

TO WIT, IN THE METRIC CLASS

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2 d\Omega^2 \qquad (20)$$

THE LAPSE FUNCTION IS, FROM B-R,

$$\begin{aligned} f(r) &= 1 - \frac{2G(r)M}{r} \\ &= \frac{B(x)}{B(x) + 2x^2}\Big|_{x=\frac{r}{G_N M}}, \end{aligned} \qquad (21)$$

WHERE

$$B(x) = x^3 - 2x^2 + \Omega x + \gamma\Omega \qquad (22)$$

FOR

$$\Omega = \frac{\tilde{\omega}}{G_N M^2} = \frac{\tilde{\omega} M_{Pl}^2}{M^2}. \qquad (23)$$

AFTER H-RADIATING TO REGIME NEAR $M_{cr} \sim M_{Pl}$, QUANTUM LOOPS ALLOW US TO REPLACE $G(r)$ WITH $G_N(1-e^{-ar})$ IN THE LAPSE FUNCTION FOR $r < r_>$, THE OUTERMOST SOLUTION OF

$$G(r) = G_N(1-e^{-ar}). \tag{24}$$

IN THIS WAY, WE SEE THAT THE INNER HORIZON MOVES TO NEGATIVE r AND THE OUTER HORIZON MOVES TO $r = 0$ AT THE NEW CRITICAL MASS $\sim 2.38 M_{Pl}$.

NOTE: M. BOJOWALD *et al.*, gr-qc/0503041, – LOOP QG CONCURS WITH GENERAL CONCLUSION.

PREDICTION: ENERGETIC COSMIC RAYS AT $E \sim M_{Pl}$ DUE THE DECAY OF SUCH A REMNANT.

PLANCK SCALE COSMOLOGY

- Bonanno and Reutersee arXiv.org:0803.2546,and refs. therein – phenomenological approach to Planck scale cosmology: STARTING POINT IS THE EINSTEIN-HILBERT THEORY

$$\mathcal{L}(x) = \frac{1}{2\kappa^2}\sqrt{-g}\,(R - 2\Lambda) \tag{25}$$

PHENOMENOLOGICAL EXACT RENORMALIZATION GROUP FOR THE WILSONIAN COARSE GRAINED EFFECTIVE AVERAGE ACTION IN FIELD SPACE$\Rightarrow$ ATTENDANT RUNNING NEWTON CONSTANT $G_N(k)$ AND RUNNING COSMOLOGICAL CONSTANT $\Lambda(k)$ APPROACH UV FIXED POINTS AS k GOES TO INFINITY IN THE DEEP EUCLIDEAN REGIME – $k^2 G_N(k) \rightarrow g_*,\ \Lambda(k) \rightarrow \lambda_* k^2$ for $k \rightarrow \infty$ IN THE EUCLIDEAN REGIME.

  – Due to the thinining of the degrees of freedom in Wilsonian field space renormalization theory, the arguments of Foot et al.(PLB664(2008)199) are obviated.

THE CONTACT WITH COSMOLOGY THEN PROCEEDS AS FOLLOWS: PHENOMENOLOGICAL CONNECTION BETWEEN THE MOMENTUM SCALE k CHARACTERIZING THE COARSENESS OF THE WILSONIAN GRAININESS OF THE AVERAGE EFFECTIVE ACTION AND THE COSMOLOGICAL TIME t, B-R SHOW

STANDARD COSMOLOGICAL EQUATIONS ADMIT THE FOLLOWING EXTENSION:

$$(\frac{\dot{a}}{a})^2 + \frac{K}{a^2} = \frac{1}{3}\Lambda + \frac{8\pi}{3}G_N\rho \tag{26}$$

$$\dot{\rho} + 3(1+\omega)\frac{\dot{a}}{a}\rho = 0 \tag{27}$$

$$\dot{\Lambda} + 8\pi\rho\dot{G_N} = 0 \tag{28}$$

$$G_N(t) = G_N(k(t)) \tag{29}$$

$$\Lambda(t) = \Lambda(k(t)) \tag{30}$$

IN A STANDARD NOTATION FOR THE DENSITY ρ AND SCALE FACTOR $a(t)$ WITH THE ROBERTSON-WALKER METRIC REPRESENTATION AS

$$ds^2 = dt^2 - a(t)^2\left(\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) \tag{31}$$

$K = 0, 1, -1 \Leftrightarrow$ **RESPECTIVELY FLAT, SPHERICAL AND PSEUDO-SPHERICAL 3-SPACES FOR CONSTANT TIME t FOR A LINEAR RELATION BETWEEN THE PRESSURE p and ρ (EQN. OF STATE)**

$$p(t) = \omega\rho(t). \tag{32}$$

FUNCTIONAL RELATIONSHIP BETWEEN THE RESPECTIVE MOMENTUM SCALE k AND THE COSMOLOGICAL TIME t IS DETERMINED PHENOMENOLOGICALLY VIA

$$k(t) = \frac{\xi}{t} \tag{33}$$

WITH POSITIVE CONSTANT ξ DETERMINED PHENOMENOLOGICALLY .

Using the phenomenological, exact renormalization group (asymptotic safety) UV fixed points as discussed above for $k^2 G_N(k) = g_*$ and $\Lambda(k)/k^2 = \lambda_*$ B-R SHOW THAT THE SYSTEM IN (30) ADMITS, FOR $K = 0$, A SOLUTION IN THE PLANCK REGIME ($0 \le t \le t_{\text{class}}$, with t_{class} a few times the Planck time t_{Pl}), WHICH JOINS SMOOTHLY ONTO A SOLUTION IN THE CLASSICAL REGIME ($t > t_{\text{class}}$) which agrees with standard Friedmann-Robertson-Walker phenomenology but with the horizon, flatness, scale free Harrison-Zeldovich spectrum, and entropy problems solved by Planck scale quantum physics.

PHENOMENOLOGICAL NATURE OF THE ANALYSIS: THE fixed-point results g_*, λ_* depend on the cut-offs used in the Wilsonian coarse-graining procedure. KEY PROPERTIES OF g_*, λ_* USED FOR THE B-R ANALYSES: they are both positive and the product $g_* \lambda_*$ is cut-off/threshold function independent.

HERE, WE PRESENT THE PREDICTIONS FOR THESE UV LIMITS AS IMPLIED BY RESUMMED QUANTUM GRAVITY THEORY, PROVIDING A MORE RIGOROUS BASIS FOR THE B-R RESULTS.

- **SPECIFICALLY, IN ADDITION TO OUR UV FIXED-PT RESULT FOR** $G_N(k) \to a^2 G_N/k^2 \equiv g_*/k^2$**, WE ALSO GET UV FIXED PT BEHAVIOR FOR** $\Lambda(k)$**: USING EINSTEIN'S EQN**

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -\kappa^2 T_{\mu\nu} \tag{34}$$

AND THE POINT-SPLITTING DEFINITION

$$\begin{aligned}\varphi(0)\varphi(0) &= \lim_{\epsilon\to 0} \varphi(\epsilon)\varphi(0) \\ &= \lim_{\epsilon\to 0} T(\varphi(\epsilon)\varphi(0)) \\ &= \lim_{\epsilon\to 0}\{: (\varphi(\epsilon)\varphi(0)) : + < 0|T(\varphi(\epsilon)\varphi(0))|0 >\}\end{aligned} \tag{35}$$

WE GET FOR A SCALAR THE CONTRIBUTION TO Λ, in Euclidean representation,

$$\Lambda_s = -8\pi G_N \frac{\int d^4k}{2(2\pi)^4} \frac{(2\vec{k}^2 + 2m^2)e^{-\lambda_c(k^2/(2m^2))\ln(k^2/m^2+1)}}{k^2+m^2}$$
$$\cong -8\pi G_N [\frac{3}{G_N^2 64\rho^2}], \quad \rho = \ln\frac{1}{\lambda_c} \tag{36}$$

with $\lambda_c = \frac{2m^2}{M_{Pl}^2}$.

For a Dirac fermion, we get -4 times this contribution.

$\Rightarrow$,

WE GET THE PLANCK SCALE LIMIT

$$\Lambda(k) \to k^2\lambda_*,$$
$$\lambda_* = \frac{1}{960\rho_{avg}}(\sum_j n_j)(\sum_j (-1)^{F_j} n_j) \tag{37}$$

where F_j is the fermion number of j, n_j is the effective number of degress of freedom of j, ρ_{avg} is the average value of ρ – see arXiv:0808.3124, Mod. Phys. Lett. A23 (2008) 3299, hep-ph/0607198.

- **All of the Planck scale cosmology results of Bonanno and Reuter, see arXiv.org:0803.2546 and refs. therein, hold, but with definite results for the limits $k^2G(k) = g_*$ and λ_* for $k^2 \to \infty$: horizon and flatness problem, scale free spectrum of primordial density fluctuations, initial entropy, etc.**

- **For reference, our UV fixed-point calculated here, $(g_*, \lambda_*) \cong (0.0442, 0.232)$, can be compared with the estimates of B-R, $(g_*, \lambda_*) \approx (0.27, 0.36)$, with the understanding that B-R analysis did not include SM matter action and that their results have definitely cut-off function sensitivity.**

- **QUALITATIVE RESULTS THAT g_* AND λ_* ARE BOTH POSITIVE AND ARE SIGNIFICANTLY LESS THAN 1 IN SIZE WITH $\lambda_* > g_*$ ARE TRUE OF OUR RESULTS AS WELL.**

- **B-R $K = 0$ solution has $\Lambda/(8\pi G_N) = 0 + \mathcal{O}(\frac{1}{t^4})$, which is too small! Usually, QFT gets this too big!**

  $\Rightarrow$ We have a chance to get the right answer? Stay tuned!

Conclusions

YFS RESUMMATION RENDERS QUANTUM GRAVITY FINITE, LOOP-BY-LOOP

- QUANTUM LOOP CORRECTIONS ARE NOW CUT OFF DYNAMICALLY.
- PHYSICS BELOW THE PLANCK SCALE ACCESSIBLE TO POINT PARTICLE QFT: (TUT)
- EARLY UNIVERSE STUDIES MAY BE ABLE TO TEST PREDICTIONS.
- MINIMAL UNION OF BOHR AND EINSTEIN
- FIRST CHECKS:

  1. MASSIVE ELEMENTARY POINT PARTCLES ARE NOT BLACK HOLES.
  2. AGREEMENT WITH BONNANNO&REUTER ASYMPTOTIC SAFETY RESULTS

     A. BLACK HOLES WITH $M < M_{cr} \sim M_{Pl}$ HAVE NO HORIZON

     B. FINAL STATE OF HAWKING RAD. IS PLANCK SCALE REMNANT, NO HORIZON – QUANTUM LOOP EFFECTS.

     C.PLANCK SCALE COSMOLOGY, $\lambda_*,\ g_*$ PREDICTED(1st principles), $\rho_\Lambda = 0$, good start.