A framework for domain-wall brane model building

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Outline

References

- Collaborators: Ben Callen, David Curtin, Aharon Davidson, Rhys Davies, Alison Demaria, Damien George, Archil Kobakhidze, Jayne Thompson, Ben Toner, Kamesh Wali.
- Our work based on foundational papers by Rubakov and Shaposhnikov (1983), Dvali and Shifman (1997), Randall and Sundrum (1999).
- I shall not give detailed references during the talk; please see our papers.

Context

Varieties of brane-world models:

- String theoretic.
- Field theoretic with brane(s) as δ-fn object(s) put in by hand. Often taken as effective low-energy outcomes of string theory, e.g. RS1.
- Completely field theoretic, with brane as soliton (domain wall, vortex, domain-wall junctions).

I shall discuss this last class. Specifically, we'll have one topologically-infinite extra dimension (like RS2) and the brane will be a domain wall.

Motivations

Why be interested in this class of brane-worlds?

- It is a bottom-up approach, contrasting with the top-down string-theory philosophy.
- It uses field theory only, including for the origin of the brane.
- Is there a phenomenologically-viable model in this class?
- Once you have a candidate for a viable model, does it solve any problems?
- All spatial dimensions on equal footing in the action.

Framework

1. Construct scalar field theory to produce required domain wall solution.
2. Dynamically localise chiral fermion zero modes (candidate quarks and leptons).
3. Dynamically localise required scalars (e.g. EW Higgs doublet).
4. Dynamically localise gauge bosons.
5. Dynamically localise gravitons.
6. See if everything works in detail, fundamentally and phenomenologically.
7. See if any mysteries are solved.

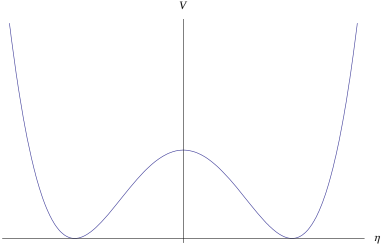
UV completion?

The 5d theory is *non-renormalisable*, even without gravity, so at the quantum level it is defined with a *UV cut-off*. This fits OK with the bottom-up philosophy: we are not trying to solve everything in one go!

Candidate UV completions:

- String theory. DW not obviously a D-brane, but maybe the effective low-energy field theory limit of some string theory admits DW solutions.
- A Lifshitz type of field theory??
- Something nobody has thought of yet.

Domain walls and kinks



We need a potential with *disconnected* and *degenerate* vacua:

$$V = \lambda(\eta^2 - v^2)^2$$

with η a scalar field.

Domain walls and kinks

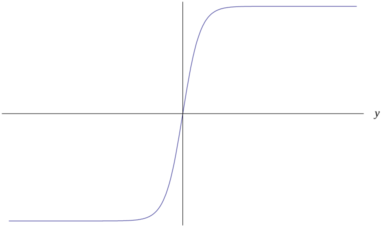
Lagrangian for $\eta(x^\mu, y)$:

$$\mathcal{L} = -\frac{1}{2}\partial_M \eta \, \partial^M \eta - V(\phi)$$

A solution is the *kink*:

$$\eta_{\text{kink}}(y) = v \tanh(\sqrt{2\lambda} v y)$$

It is topologically stable.



Fermion localisation

Let $\Psi(x, y)$ be 5d fermion Yukawa coupled to background scalar field $b(y)$. It obeys 5d Dirac Eq:

$$i\Gamma^M \partial_M \Psi - b(y)\Psi = 0$$

where $\Gamma^M = (\gamma^\mu, -i\gamma_5)$.

Do mode decomposition (generalised KK expansion):

$$\Psi(x, y) = \sum_m \left[f_L^m(y)\psi_L^m(x) + f_R^m(y)\psi_R^m(x) \right]$$

The ψ's are 4d fermions, forced to obey 4d Dirac Eq:

$$i\gamma^\mu \partial_\mu \psi_{L,R}^m = m\psi_{R,L}^m$$

Fermion localisation

The mode functions $f^m_{L,R}$ then obey the Schrödinger-like equations

$$-{f^m_{L,R}}'' + W_\mp f^m_{L,R} = m^2 f^m_{L,R}$$

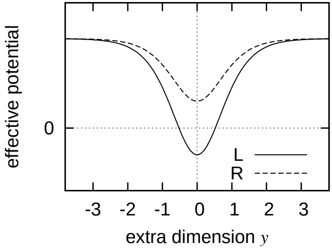
with effective potentials

$$W_\mp(y) = b(y)^2 \mp b'(y).$$

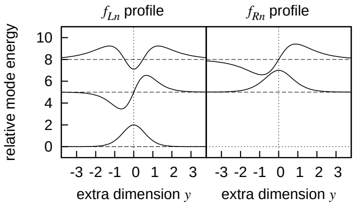
At points y_0 s.t. $b(y_0) = 0$, you get a *localised chiral zero mode*. The chirality depends on whether $b(y)$ slopes up (LH) or down (RH).

These are our candidate quarks and leptons.

Fermion localisation



Fermion trapping potential



Fermion modes

Fermion localisation

The profile for the chiral zero mode is:

$$f(y) \propto e^{-\int_{y_0}^{y} b(y')dy'}$$

The b's depend on the Yukawas, so the f's are *exponentially sensitive* to them. This is the key to getting fermion mass hierarchies (see later).

Mode functions are normalised as per $\int_{-\infty}^{+\infty} f(y)^2 dy = 1$ to make 4d field kinetic terms conventionally normalised.

Spin-0 boson localisation

This proceeds in a very similar way to fermions. I'll omit the details here.

One important outcome: a localised 4d scalar can have a tachyonic mass-squared. Hence we can have SSB inside the wall. We'll use this for the electroweak Higgs doublet.

Graviton localisation

A reasonably trivial modification of type-2 Randall-Sundrum:

- Effective “volcano” potential describes graviton localisation.
- 4d graviton zero mode solution.
- No mass gap to graviton KK excitations, but OK because their mode functions are very suppressed inside the wall (tunnelling through a barrier).
- The warped metric turns the effective localisation potentials for fermions and scalars from wells into volcanoes: no mass gap, but OK for same reason as gravitons.

Gauge field localisation

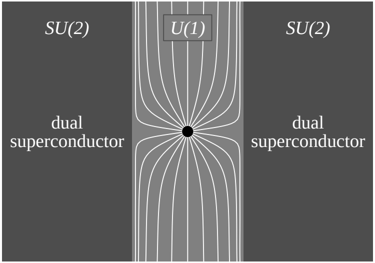
This is the trickiest issue. I shall discuss a proposal called the Dvali-Shifman mechanism. Its validity is not established, but there are plausibility arguments. Assuming it works allows model building to progress!

It requires:

1. The unbroken symmetry inside the wall to be a subgroup of the bulk symmetry.
2. The bulk to be in confinement phase.

Unlike other types of field localisation, this mechanism relies on non-perturbative QFT rather than classical FT.

Gauge-field localisation



The confining bulk repels the field lines of a source inside the wall. The large distance gauge field behaviour inside the wall is effectively dimensionally reduced by 1.

The background DW

Dvali-Shifman tells us that we should embed $SU(3) \times SU(2) \times U(1)$ in a larger group that breaks to it inside the wall.

The minimal sensible choice is $SU(5)$.

We use an $SU(5)$ singlet scalar η to produce a kink, and an $SU(5)$ adjoint χ to break $SU(5)$ to the SM inside the wall.

Write $\chi = \sum_a T^a \chi_a$, where T's are $SU(5)$ generators in the fundamental. If the component χ_1 corresponding to the hypercharge generator Y condenses, then $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)_Y$.

The background DW

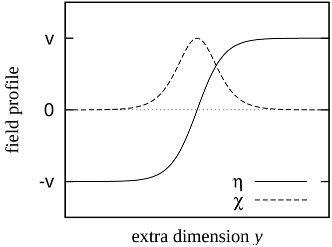
You write a Higgs potential, arrange the global minima

$$\langle\eta\rangle = \pm v,\ \langle\chi\rangle = 0,$$

use them as boundary conditions, solve the Euler-Lagrange equations to get, e.g.

$$\eta(y) = v\tanh(ky),$$

$$\chi_1(y) = A\,\mathrm{sech}(ky).$$



This simple analytical solution holds on a certain parameter slice. Off that slice, similar solutions exist but must be obtained numerically.

Fermion localisation

Next, you introduce 5d fermions

$$\Psi_5 \sim 5^*, \ \ \Psi_{10} \sim 10, \ \ N \sim 1$$

and you Yukawa couple them to η and χ:

$$\begin{aligned} Y_{DW} &= h_{5\eta}\overline{\Psi}_5\Psi_5\eta + h_{5\chi}\overline{\Psi}_5\chi^T\Psi_5 \\ &+ h_{10\eta}\mathrm{Tr}(\overline{\Psi}_{10}\Psi_{10})\eta - 2h_{10\chi}\mathrm{Tr}(\overline{\Psi}_{10}\chi\Psi_{10}) \\ &+ h_{1\eta}\overline{N}N\eta. \end{aligned}$$

The background fields you use in the 5d Dirac Eq. are:

$$b_{nY}(y) \equiv h_{n\eta}\eta(y) + \sqrt{\frac{3}{5}}\frac{Y}{2}h_{n\chi}\chi_1(y).$$

SM components of different hypercharge Y couple to different linear combinations of $\eta(y)$ and $\chi_1(y)$. Fermions are *split*, but not arbitrarily.

EW symmetry breaking

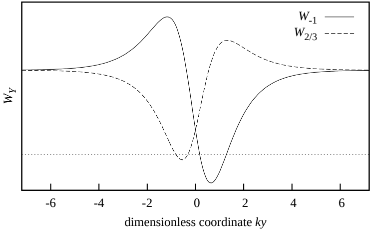
Now introduce a scalar $\Phi \sim 5^*$ containing the weak doublet Φ_w and a coloured scalar Φ_c. Yukawa couple it to fermions in the usual way.

You do a mode decomposition, and are interested in the lowest modes:

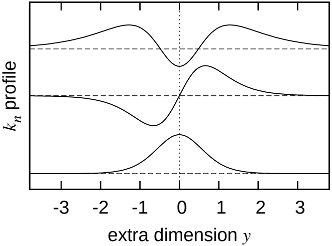
$$\Phi_{w,c}(x, y) = p_{w,c}(y)\phi_{w,c}(x)$$

You write the Higgs potential, plug the above into the Euler-Lagrange Eqs., get effective Schrödinger Eqs. for the profiles $p(y)$.

EW symmetry breaking



Scalar trapping potentials



Generic scalar modes

The p_w well is deeper (due to parameter region chosen) and gets a negative evalue m_w^2 triggering spontaneous EW symmetry breaking.

Fermion spectra

I now report some preliminary results from work by Ben Callen on fitting the model to the observed quark and lepton masses, including neutrinos.

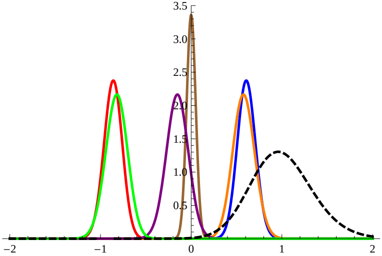
A 4d Yukawa coupling is of the form:

$$h\left[\int dy f_L(y) f_R(y) p(y)\right]\overline{\psi}_L(x)\psi_R(x)\phi(x).$$

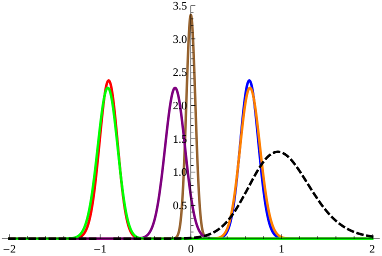
The 4d Yukawa coupling constant is equal to the 5d Yukawa multiplied by an overlap integral of profile functions, which themselves depend on Yukawas in a complicated way.

The profiles are *exponentially sensitive* to the Yukawa coupling constants. Searching the parameter space is numerically intensive. We have been proceeding by trial-and-error. Multiple viable regions exist.

Fermion spectra – no mixing



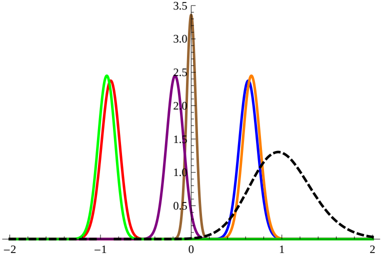
3rd gen profiles



2nd gen profiles

Brown - the right handed neutrinos; Red - the left handed lepton doublets; Blue - right handed down, strange, and bottom quarks; Green - right handed electron, muon and tau; Orange - right handed up, charm, and top quarks; Purple - the left handed quark doublets; Black dashed - the Electroweak Higgs.

Fermion spectra – no mixing



1st gen profiles

All EW Yukawas have been set as equal!

Fermion mass differences *entirely* driven by profiles through coupling to the DW background. The masses can be fitted well, including very light Dirac neutrinos.

Spread used in fermion-DW Yukawas is less than order-of-mag. Fermion mass spread is nevertheless 14 orders-of-mag.

Higgs-induced proton decay suppression

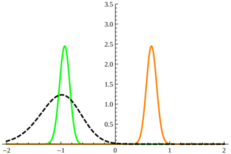
This process proceeds via the Yukawa terms

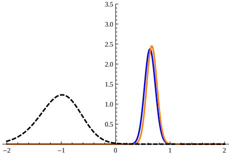
$$\overline{u}_R (e_R)^c \phi_c^* \qquad \text{and} \qquad \overline{d}_R (u_R)^c \phi_c .$$

For the *same* region of parameter space that fits the masses, the effective 4d Yukawa couplings constants are, respectively, about

$$10^{-31} \qquad \text{and} \qquad 1.$$

The proton partial lifetime goes below the experimental bound for ϕ_c mass greater than just a few GeV!.





Summary

- There is a framework for potentially viable domain-wall brane models.
- The dynamical localisation of fermions, scalars and gravitons is well understood.
- The dynamical localisation of gauge bosons is not well understood (in field theory). The Dvali-Shifman proposal allows model building to proceed readily. Its validity is an open question.
- Combining the mechanisms, one can construct a 5d SU(5) model that can alleviate the fermion hierarchy problem and has slow Higgs-induced proton decay.
- Open problems: gauge coupling constant unification, gauge-boson-induced proton decay, gauge hierarchy, dark matter.