A BEAUTIFUL MOND ?

G B Tupper
U C T

“Missing mass problem”

- Zwicky (1933) Cluster Dymamics
- 1970’s: Flattened Galactic Rotation Curves

+

Standard **M**odel of **P**article **P**hysics

- *Beyond* **SMPP :** *Dark Matter*

or

__Mo__dified __N__ewtonian __D__ynamics (**MOND**) ?

MOND

- Milgrom (1983)[1]

  Modify inertia, minimum *acceleration* a_0

$$\vec{F} = m\,\vec{a}\,\mu\left(\left|\vec{a}\right| / a_0 \right)$$

$$\mu(x) \approx \frac{x}{1+x} \qquad a_0 \approx H_0 \approx \Lambda_{DE}$$

MOND

- Bekenstein & Milgrom (1984)[2]
- Keep $\vec{a} = \vec{\nabla}\varphi$
- Modify Poisson equation

$$\vec{\nabla} \bullet \left(\mu\left(\frac{\left|\vec{\nabla}\varphi\right|}{a_0} \right) \vec{\nabla}\varphi \right) = 4\pi G_N \rho$$

$$\updownarrow$$

$$\mu(x) = \varepsilon\left(x^2\right) + x^2 \varepsilon'\left(x^2\right)$$

$$S_{MOND} = \int d^3x \left[-\rho\varphi - \frac{\left|\vec{\nabla}\varphi\right|^2}{8\pi G_N} \varepsilon\left(\frac{\left|\vec{\nabla}\varphi\right|^2}{a_0^2} \right) \right]$$

MOND

$$\mu(x) = \frac{x}{1+x} \Leftrightarrow \varepsilon\left(x^2\right) = 1 + \frac{2}{x^2}\left[\ln(1+x) - x\right]$$

$$\approx \frac{2}{3}x,\ x << 1$$

Relativistic MOND?

- Bekenstein (2004)[3] : TeVeS

  *Bi*metric theory $g_{\mu\nu} = e^{-2\phi}\tilde{g}_{\mu\nu} + 2\sinh(2\phi)A_\mu A_\nu$

- Matter frame **Te**nsor $g_{\mu\nu}$
- Einstein-Hilbert $\tilde{g}_{\mu\nu}$
- Constrained **Ve**ctor $A_\mu A_\nu \tilde{g}^{\mu\nu} = 1$
- **S**calar ϕ
- Zlosnik et al (2006): constraint $\Rightarrow e^{-2\phi} = A_\mu A^\mu$

Relativistic MOND?

- Zlosnik et al (2006)[4]: *Nonlinear Aether* MOND
- **One** metric $g_{\mu\nu}$
- Timelike vector A_μ
- Action $(c_1 = 1 = -c_3, c_2 = 0)$ $S = S_{EH} + S_A + S_{matter}$

$$S_A = \int d^4x\sqrt{-g}\left[4a_o^2\left(\sqrt{\kappa} - \ln\left(1+\sqrt{\kappa}\right)\right) + \lambda\left(A_\mu A^\mu - 1\right)\right]$$

$$\kappa = -\frac{F_{\mu\nu}F^{\mu\nu}}{2a_0^2} \quad Newtonian \Rightarrow \vec{E} \approx -\vec{\nabla}\varphi, \vec{B} \approx 0$$

$$\kappa << 1 \Rightarrow \Im_A \approx \kappa/2 - \kappa^{3/2}/3$$

Relativistic MOND?

- Milgrom Bimetric (2009)[5]

MOND

- Bekenstein & Milgrom (1984)[2]

$$S_{MOND} = \int d^3x \left[-\rho\varphi - \frac{\left|\vec{\nabla}\varphi\right|^2}{8\pi G_N} \varepsilon\left(\frac{\left|\overrightarrow{\nabla\varphi}\right|^2}{a_0^2} \right) \right]$$

"like nonlinear electrodynamics and **QCD.**"

QCD

- Pagels & Tomboulis (1978)[6]: quantum field theory, renormalization <u>breaks</u> *conformal invariance* ; **eff**ective action

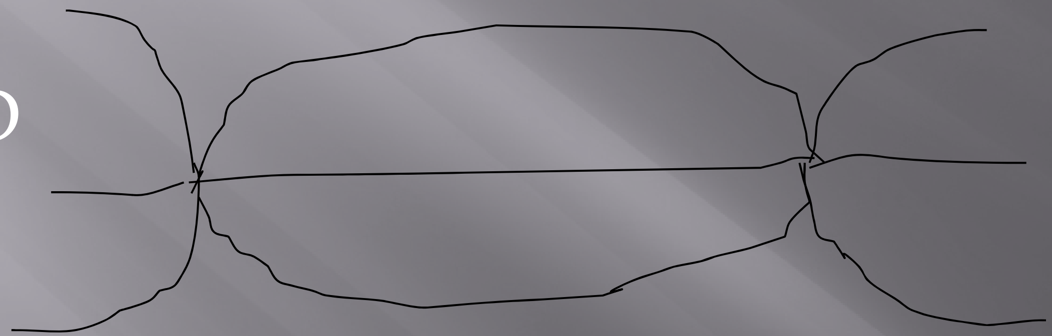
$$\Im_{QCDeff} = -\frac{\vec{F}^{\mu\nu} \bullet \vec{F}_{\mu\nu}}{4\bar{g}_3^{-2}(t)} \qquad t = \frac{1}{4}\ln\left(\frac{\vec{F}_{\mu\nu} \bullet \vec{F}^{\mu\nu}}{\mu_{QCD}^4}\right) = \int_{g_3}^{\bar{g}_3} \frac{dg_3}{\beta(g_3)}$$

- Asymptotic freedom $\beta(g_3) \cong -bg_3^3$ small g
- Infrared slavery, confinement

QCD

Linear potential $\Leftrightarrow \beta(g_3) \approx -g_3$

QED



QCD flux tube

QCD

- Interpolate $\beta(g_3) = -\frac{b g_3^3}{1 + b g_3^2}$

$$\frac{1}{2}\ln\left(\frac{\vec{F}_{\mu\nu} \bullet \vec{F}^{\mu\nu}}{\Lambda_{QCD}^4}\right) = \frac{1}{b\bar{g}^2} - \ln\left(\bar{g}^2\right)$$

Large field $\Leftrightarrow$ weak coupling

small field $\Leftrightarrow$ strong coupling

IR $\quad \Im_{QCDeff} \cong -\frac{\Lambda^4_{QCD}}{4}\left(\frac{\vec{F}_{\mu\nu} \bullet \vec{F}^{\mu\nu}}{\Lambda^4_{QCD}}\right)^{3/2}$

Compare

QCD

- Gluons self interact
- Renormalizable
- Asymptotically free
- Local gauge invariance
- Broken scale invariance, effective action

GRAVITY

- Gravitons self interact
- Nonrenormalizable, effeftive field theory [7]
- Asymptotically *safe* *[8]*
- *General coordinate invariance*
- ??

"**D**ielectric **I**nfrared **M**odified **N**onlinear **D**ynamics"

DIMOND

Require

- Pure gravity
- Reduce to **G**eneral **R**elativity for 'strong fields'
- *Natural* MONDian limit for weak fields

not $f(\Re)$ as Ricci $\Re \approx \overrightarrow{\nabla}^2 \varphi$

- Note GR has **G**eneral **C**oordinate **I**nvariance *and*

Local **L**orentz **I**nvariance

DIMOND

- Fermions (matter) require tetrad field e_{μ}^{m}

$$g_{\mu\nu} = e_{\mu}^{m} e_{\nu}^{n} \eta_{mn}$$

- GR has hidden LLI $\Re = \Re(\Gamma)$

  Christoffel connection torsionless

$$T_{\mu\nu}^{\alpha} = \Gamma_{\nu\mu}^{\alpha} - \Gamma_{\mu\nu}^{\alpha} = 0$$

- Weitzenbock connection $\dot{\Gamma}_{\mu\nu}^{\alpha} \equiv e_{a}^{\alpha} e_{\nu,\mu}^{a}$

  nonzero *torsion* $\dot{T}_{\mu\nu}^{\alpha} \neq 0$

  zero curvature $\dot{\Re} = \Re(\dot{\Gamma}) = 0$

  Weitzenbock spacetime: absolute parallelism

Torsion Gravity[9]

- Contortion *tensor* $K^{\alpha}_{\mu\nu} = \dot{\Gamma}^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\mu\nu}$
- Denote $K \bullet K \equiv g^{\mu\nu}(K^{\beta}_{\mu\alpha} K^{\alpha}_{\nu\beta} - K^{\alpha}_{\mu\nu} K^{\beta}_{\alpha\beta})$
- Up to a "surface term"

$$S_{EH} = \frac{-1}{16\pi G_N} \int d^4x \sqrt{-g}\,\Re = \frac{1}{16\pi G_N} \int d^4x\, e K \bullet K$$

"Teleparallel equivalent of GR"

DIMOND

- $K \bullet K$ Is GCI scalar *but*

$$\delta_{LLT} K \bullet K = 4 - divergence \neq 0$$

- *Hypothesis: (infrared) effects in quantum gravity break LLI (like conformal invariance in QCD) but preserve GCI (like gauge invariance in QCD)*

DIMOND

- Propose *effective* action:

$$S_{DIMOND} = \frac{1}{16\pi G_N} \int d^4 x e K \bullet K \varepsilon \left(\frac{-K \bullet K}{2a_0^2} \right)$$

- GR limit *automatic*
- Newtonian MOND limit *automatic*

“Conclusions”

- Is DIMOND ‘beautiful’ ?
- Can one *derive* breaking of LLI and so effective action?
- What about cosmology – especially ‘dark energy’ ? [10]

References

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