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NONEXTENSIVITY IN A DARK MAXIMUM ENTROPY LANDSCAPE

M. P. LEUBNER

Institute for Astro- and Particle Physics

University of Innsbruck, Austria

TOPICS

(1) FUNDAMENTALS OF NONEXTENSIVE STATISTICS

(2) ENTROPY BIFURCATION AND DUALITY

(3) CONSEQUENCES OF NONEXTENSIVITY

- (1) Non-thermal equilibria in thermostatistics - plasma
- (2) Gravitational equilibria in self-gravitating systems - DM
- (3) Nonextensive negative pressure domain – DE
- (4) Discrete cosmic inhomogeneity scales

(4) Summary

Four fundamentally different domains in nonxtensive statistics depending on sign and value of the governing entropic index

NATURE IS NONEXTENSIVE

ANY MEMBER OF AN ENSEMBLE OF PARTICLES

↓

gravitational / electromagnetic / strong / weak …

long-range interactions → correlations, coupling

TWO EXTREME CASES

(1) crystal – max order → min entropy, exact geometry
(2) therm gas – min order → max entropy, BGS statistics

NATURE SOMEWHERE BETWEEN

NATURE IS COMPLEX → STATISTICS

EXTENSIVE FUNDAMENTALS

Statistical Mechanics → adoption of specific Entropy Functional S

↓

shortcut for vast, detailed microscopic information stored in system
+ connection to macroscopic behavior

no systematic context determining S

Boltzmann – Gibbs proposed fundation of standard statistics
Shannon: descrete form
entropic form for ergodic systems

$$S = - V p_i \ln p_i$$

Systems visiting with equal probability all allowed states
Chaotic systems → molecular chaos hypothesis
Extensive systems: no correlations, members independent

BUT: nonergodicity is the generic case for complex systems

NONEXTENSIVE FUNDAMENTALS

Nonextensive Statistics → what is S for non-ergodic systems

$$S_\kappa = \kappa\left(\sum p_i^{1-1/\kappa} - 1\right)$$

p_i...probability of i-th microstate
S extremized for equiprobabilit

$$\sum p_i = 1$$

$\kappa \to \infty$ $S = -\sum p_i \ln p_i$

Tsallis, 1988 q-entropy conjecture

transformation $\kappa = 1/(1-q)$

Leubner, 2002 $-\infty \leq \kappa \leq \infty$

(1) symmetry in entropic index

(2) link astrophysical nonext PDFs

$$S_\kappa(A+B) = S_\kappa(A) + S_\kappa(B) + \frac{1}{\kappa} S_\kappa(A) S_\kappa(B)$$

$\kappa > 0$ ... superextensive
$\kappa < 0$ ... subextensive

DUALITY + less organized state → entropy increase
- higher organized state → entropy decrease

Nonext statistics - applications

Chemistry, Biology, Economics, Linguistics, Medicine, Geophysics, Social & Cognitive Science, Computer Science, Information Theory…

Physics:

- Astrophysical plasma distributions in velocity space
- PDF's of differences of fluctuations in plasma turbulence
- DM and plasma density distributions of self-gravitating systems
- Distribution of peculiar velocities of spiral galaxies
- Cosmic microwave background radiation
- Nonextensive dark energy model - chaotic scalar fields
- Transverse momenta distribution in hadronic jets, e^-e^+ annihilation
- Diffusion of charm quarks in quark-gluon plasma
- Hierarchy of discrete structure scales in the universe

etc. etc.….

EXTENSIVE / NONEXTENSIVE

EXTENSIVE SYSTEMS

particles independent feo
no interactions/correlations
Euclidian space time

standart BGS statistics → logarithmic entropy measure additive

Maxwell DF $f_M(E, \Phi)$

NONEXTENSIVE SYSTEMS

particles not independent feo
long-range forces / memory,
correlations, fractal space time

Nonextensive statistics → generalized power-law entropy pseudoadditive

power-law DF $f_\kappa(E, \Phi)$

DUALITY

(A) Duality of NEXT equilibria

TWO FAMILIES (κ, κ′) OF STATIONARY STATES (Karlin, 2002)

thermodynamic equilibrium (κ) » ⟹ maximum entropy state

kinetic equilibrium (κ′) » ⟹ zeros of the collision integral

constraint for the nonextensive equilibrium index

⇓

nonextensive thermodynamic equilibria K > 0

nonextensive kinetic equilibria K′ < 0

with condition κ′ = - κ DUALITY

limiting BGS state for κ = ∞ : SELF - DUAL » extensivity

the extensive κ = ∞ BGS state bifurcates for finite values of κ

(B) Duality of NEXT heat capacity

TWO FAMILIES OF HEAT CAPACITIES (Almeida, 2001)

Virial theorem

2K + U = 0

E = K + U = - K

$K = 3/2\, Nk_B T$

C = dE/dT

$C = -\,3/2\, Nk_B < 0$

$$\frac{d}{dE}\left(\frac{1}{\beta}\right) = \frac{1}{\kappa}$$

κ…related to energy derivative of heat bath

$$\frac{1}{\beta} = kT$$

$$\frac{d}{dE}\left(\frac{1}{\beta}\right) = \begin{cases} >0 \ldots \kappa > 0 \\ 0 \ldots \kappa = \infty \\ <0 \ldots \kappa < 0 \end{cases}$$

positive HC

infinite HC

negative HC

K > 0……..positive HC … thermodynamic systems
K = ∞…ideal heat bath, loss/gain energy without change of T
K < 0 …..negative HC … self-interacting systems

The BGS self-dual state (κ = ∞) bifurcates for finite κ »

IDENTIFICATION κ > 0 … thermodynamic state … plasma
κ < 0 … self-interacting state … DM

FROM ENTROPY GENERALIZATION TO PDFs

S_κ... extremize entropy under conservation of mass and energy

variational problem $\quad\quad \delta S - \alpha\, \delta m - \beta\, \delta E = 0$

α,β... Lagrage multiplieres

POWER-LAW DISTRIBUTIONS, BIFURCATION n ‹ 0

halo n > 0

$$f_{hc} = B_{hc}\left(1 + \frac{v^2}{\kappa\sigma^2}\right)^{-\kappa}$$

core n < 0

⇙ ⇘

incorporate sign of κ in distribution

$$B_c = \frac{N}{\pi^{1/2} v_{th}} \frac{\Gamma(\kappa)}{\kappa^{1/2}\Gamma(\kappa - 1/2)}$$

normalization

n » | n |

$$B_c = \frac{N}{\pi^{1/2} v_{th}} \frac{\Gamma(\kappa + 3/2)}{\kappa^{1/2}\Gamma(\kappa + 1)}$$

σ^2...variance, mean energy

2nd moments and constraints

$$f_c = B_c(1+\frac{v^2}{|\kappa|\sigma^2})^{-|\kappa|}$$

halo

0 ¸ n ¸ ∞

$$f_h = B_h(1-\frac{v^2}{|\kappa|\sigma^2})^{|\kappa|}$$

core

different
generalized 2nd moments

$$p = \frac{Nm}{2}\int v^2 f_{\kappa} dv$$

identical for temperatures

as compared to BGS state

n **> 0**

$$p_\kappa = \frac{\kappa}{\kappa - 3/2} p_B$$

increased
and

0 ? n ? **3/2**

neg pressure

n **< 0**

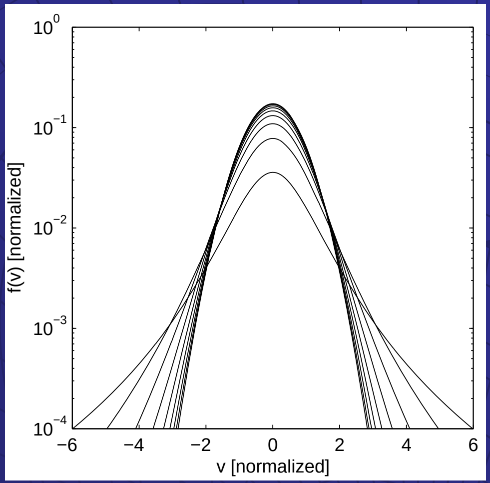
$$p_\kappa = \frac{\kappa}{\kappa + 3/2} p_B$$

decreased
and

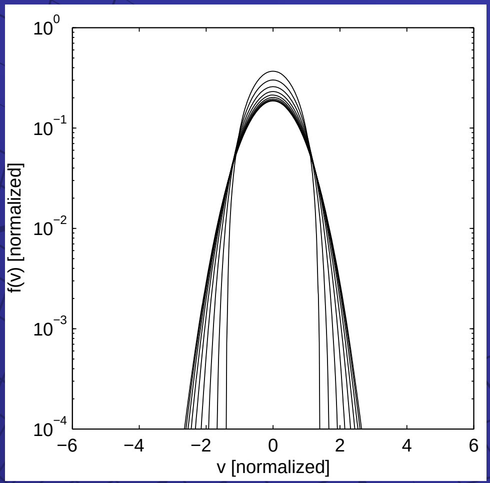
v = σ ô κ

cutoff

KAPPA - DEPENDENT CHARACTERISTICS



halo, n > 0

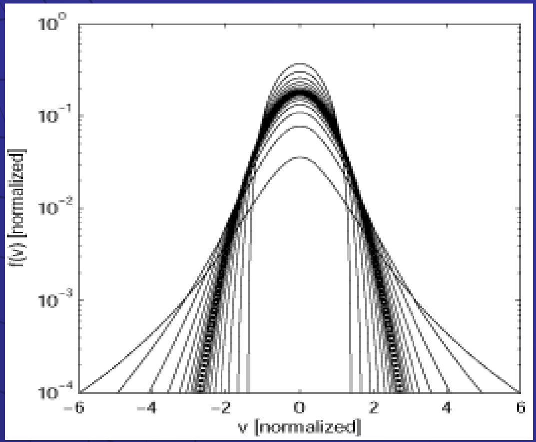


core, n < 0

nonextensive distribution family and Maxwellians

$\kappa = 3 \dots \infty$

BI - KAPPA PDF
core-halo superposition



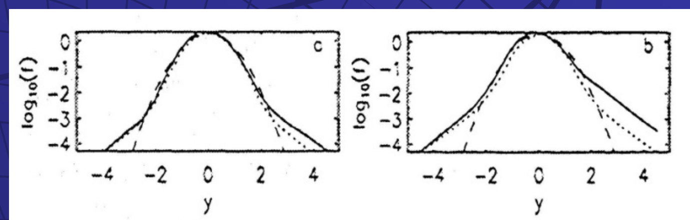
(1) Electromagnetic Interactions

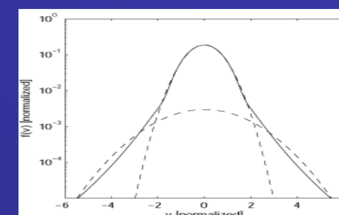
K > 0, C > 0, p > 0

BEYOND: REPLACE STANDARD MAXWELLIANS BY NEXT DISTR. FAMILY

a) Energy distributions in astrophysical plasmas

e^- VDF observed

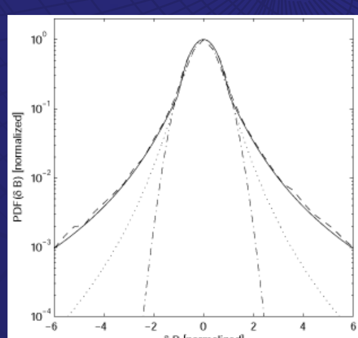




nonext theory

Leubner, 2002, 2004

b) PDF's in astrophysical plasma turbulence



$$\delta X = X(t+\tau) - X(t)$$

amplitude

PDFs of differences of plasma or magnetic field fluctuations

Leubner & Voros, 2005, 2006

(2) Grav. Equ. - DM/plasma density distributions

STANDARD CONTEXT: GRAV. INFALL MODELS; PRESSURE BALANCE

BEYOND: EXTREMIZE GENERALIZED ENTROPY OF NEXT STATISTICS

hot gas » elm. interacting, fully ionized plasma, thermodyn. equilibrium

DM halo » self grav., weakly interacting particles, dynamical equilibrium

extremize S in grav. Potential Ψ
conservation of mass and energy

$$S_\kappa = \kappa\left(\sum p_i^{1-1/\kappa} - 1\right)$$

» energy distribution

$$f^{\pm}(E_r) = B^{\pm}\left[1 + \frac{1}{\kappa}\frac{v^2/2 - \Psi}{\sigma^2}\right]^{-\kappa}$$

bifurcation: κ > 0 κ < 0

integration over v

$$\rho^{\pm} = \rho_0\left[1 - \frac{1}{\kappa}\frac{\Psi}{\sigma^2}\right]^{3/2-\kappa}$$

limit κ = ∞ : extensive exponential – density profile

$$\rho = \rho_0 \exp(\Psi/\sigma^2)$$

Nonextensive spatial density variation

K < 0, C < 0, p > 0

$$\Delta\Psi = -4\pi G\rho$$

combine

$$\rho^{\pm} = \rho_0 \left[1 - \frac{1}{\kappa}\frac{\Psi}{\sigma^2}\right]^{3/2-\kappa}$$

$$\frac{d^2\rho}{dr^2} + \frac{2}{r}\frac{d\rho}{dr} - \left(1 - \frac{1}{3/2-\kappa}\right)\frac{1}{\rho}\left(\frac{d\rho}{dr}\right)^2 - \frac{4\pi G\left(3/2-\kappa\right)}{\kappa\sigma^2}\rho^2\left(\frac{\rho}{\rho_0}\right)^{-1/\left(3/2-\kappa\right)} = 0$$

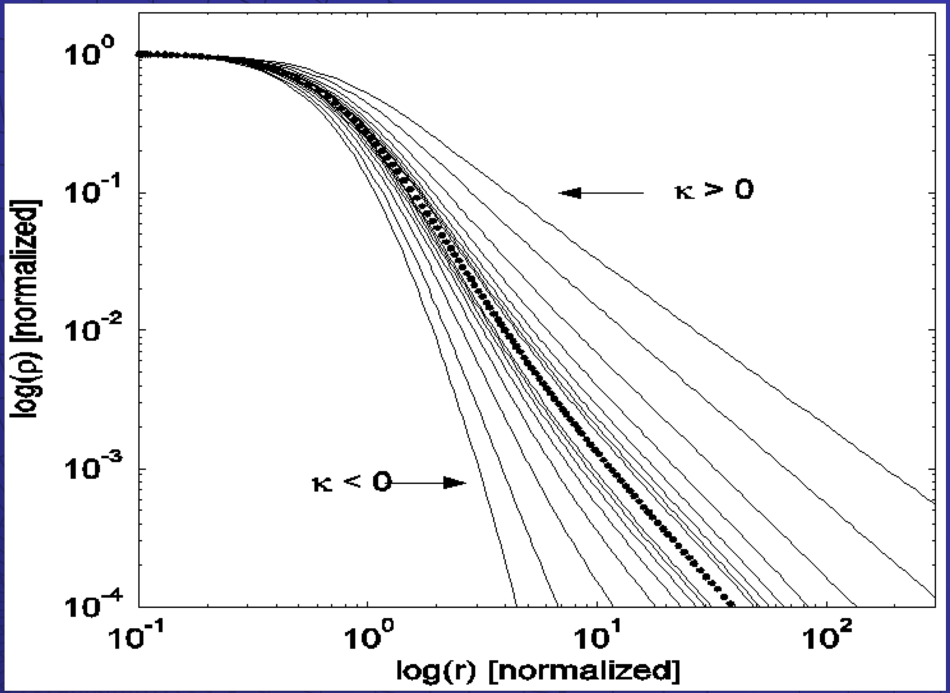
Ψ = Ψ(r)

Leubner, 2005, 2006

ρ(r) … radial density distribution of spherically symmetric hot plasma (n > 0) and dark matter (n < 0)

κ = ∞ … BGS selfduality, conventional isothermal sphere

Non-extensive family of density profiles



Nonextensive family of density profiles $\rho^{\pm}(r)$, $\kappa = 3 \ldots 10$

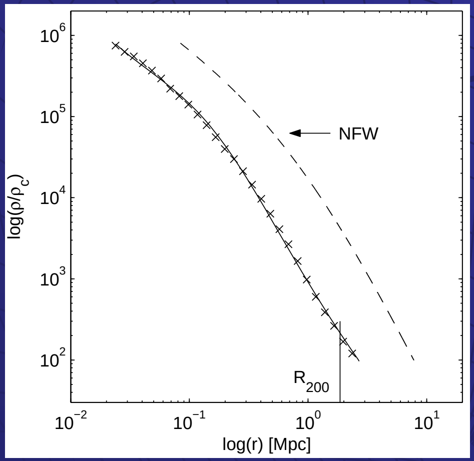
Convergence to the selfdual BGS solution $\kappa = \infty$

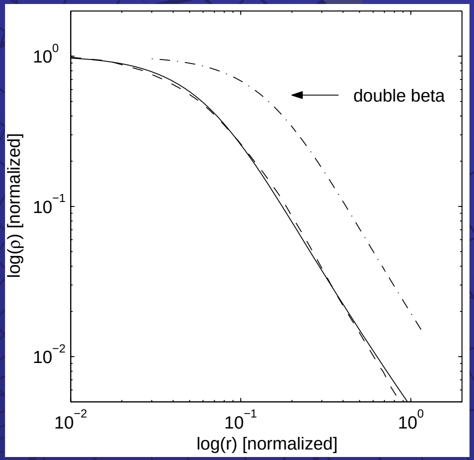
Testing Nonextensive Density Distributions on Simulations, Observations

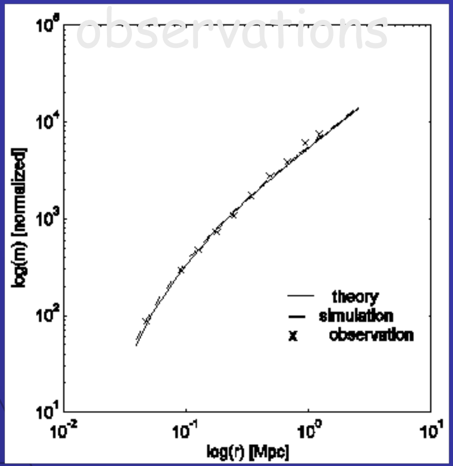
DM simulations
K < 0

Hydro simulations
K > 0

Observations
K < 0







Kronberger, Leubner, van Kampen A&A, 2006

Integrated mass profiles
Pointecouteau et al., A&A 2005

(3) Nonextensivity in a dark energy landscape

Universe currently in accelerated expansion

73% DE - 23% DM - 4% ordinary matter

STANDARD DARK ENERGY MODELS:

(1) Vacuum energy of self-interacting scalar field
potential energy generates a cosmological constant
(2) Quintessence models with slowly evolving scalar fields and non-trivial equation of state
(3) String theoretical candidates of scalar fields and consequences
(4) Exotic forms of matter

BEYOND: NEXT STAT + SCALAR FIELD POTENTIAL

Scale factor: dark energy, matter, radiation

3 components (j) in homogeneous, isotropic universe - adiabatic expansion:

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho_{DE} + 3p_{DE} + \rho_m + \rho_r + 3p_r)$$

ρ...energy density, p...pressure

$$H^2 = (\dot{R}/R)^2$$

$$\frac{d\rho_j}{dt} = -3H(\rho_j + p_j)$$

conservation of energy

Equation of state for dominant species j $w_j = \langle p_j \rangle / \langle \rho_j \rangle$ = const

$$\rho_j \sim R^{-3(1+w_j)}$$

$\rho_m \sim R^{-3}$ $w_m = 0$

$\rho_r \sim R^{-4}$ $w_r = 1/3$

$\rho_{DE} \sim$ const $w_{DM} = -1$,

class. vacuum energy condition

condition

$$\frac{3p_{DE}}{\rho_{DE} + \rho_m} < -1$$

NEXT chaotic scalar fields as DE model

consider chaotic strongly self-interacting scalar fields

associate chaotic behavior with vacuum fluctuations

probability distribution of fluctuating field

$$p(\phi) = \frac{1}{\pi\sqrt{1-\phi^2}}$$

Beck, 2004

associate generalized NEXT distributions

$$p(E) \sim (1 + \frac{1}{\kappa}\beta E)^{-\kappa}$$

identify $E = m\Phi^2/2$ and $\kappa = 1/2$

$\beta^{-1} = m$ … thermal energy coincides with mass

satisfy property of NEXT DE gas: $w = -1$?

Nonextensive dark energy domain

$0 < \kappa < 3/2, \; C > 0, \; p < 0$

2nd moment $p \sim \rho \int v^2 f(v) dv$ $\Longrightarrow$ $p_{DE} = \dfrac{2\kappa}{\kappa - 3/2} \rho_{DE}$

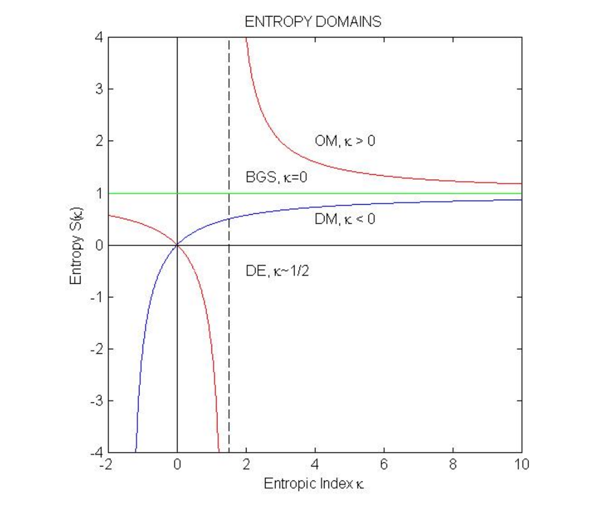
→ DE equation of state (Λ) $w \sim p_{DE} / \rho_{DE} \sim -1$

→ density parameter $\Omega_{DE} = \dfrac{\rho_{DE}}{\rho_{DE} + \rho_m + \rho_r} \sim 0.75$

entropic index κ specifies degree of correlations / heat capacity

- dark energy behaves like ordinary gas ($C > 0$)
- subject to high degree of correlations ($\kappa \sim 1/2$)
- satisfying equation of state $w \sim p_{DE} / \rho_{DE} \sim -1$

ENTROPIC DOMAINS



$$S_\kappa = \kappa\left(\sum p_i^{1-1/\kappa} - 1\right)$$

Negative entropy:

Entropy that system exports to keep own entropy low

Schrödinger, 1943

Green: BGS extensive statistics, ergodic, S(κ) = S_M = const, κ = ∞

Red: nonextensive thermostat, non-ergodic, S(κ) > S_M, κ > 0, reduced order

Blue: nonextensive kin. stat, DM, non-ergodic, S(κ) < S_M, κ < 0, increased order

0 < κ < 3/2 : DE domain, chaotic scalar fields, S(κ) < S_M, κ ~ 1/2, m ~ 10e-38 m_e

(4) Discrete cosmic inhomogeneity scales

STANDARD CONTEXT: NONE

BEYOND: NEXT STAT + NETWORK SCIENCE – DIMENSIONLESS

emerging, highly interdisciplinary research area
tradition in
graph theory, discrete mathematics, sociology,
information /communication theory, biology, physics…

undirected network of network ensembles with N nodes and L links
Common context: node entropy, link entropy

(1) Entropy per node in a network ensemble N_i $S_i = 1/N_0 \ \ln N_i$
(Hartley information measure – ident BGS)

(2) Probability of given link (i) $p_i = L \ / \ (N_0(N_0-1)/2 \ )$

(3) Perform nonextensive generalization with $N_0 \gg 1$

Nonextensive network generalization

$$S_\kappa = \kappa(\sum p_i^{1-1/\kappa} - 1)$$

Let members of cluster merge ⟹ links within / between subclusters

N_0^2 … total number of links in entire original cluster network
n^2 …… total number of links within each subcluster network
N^2 …. total number of links between all subclusters as closed units

condition: subclusters obey closed network
nodes correspond to particles - node conservation: $N*n = N_0$

HOW DOES ENSEMBLE RE-ARRANGE ⟹ EXT. SUM OF LINKPROB.

$S_\kappa = \kappa \{[(N n^2 + N^2)/N_0^2]^{1-1/\kappa} - 1 \}$ ⟹ Ext.

LINK ENTROPY

Leubner, M. P., 2000, 2002, 2010

$N = 2^{-1/3} N_0^{2/3}$

no dependence on κ - connected or not

Recursion context - network hierarchy

(1) Hierarchical tree - nested generalization

$$N_{i+1} = 2^{-1/3} N_i^{2/3}$$

(2) Introducing physics (m, r) through natural invariant

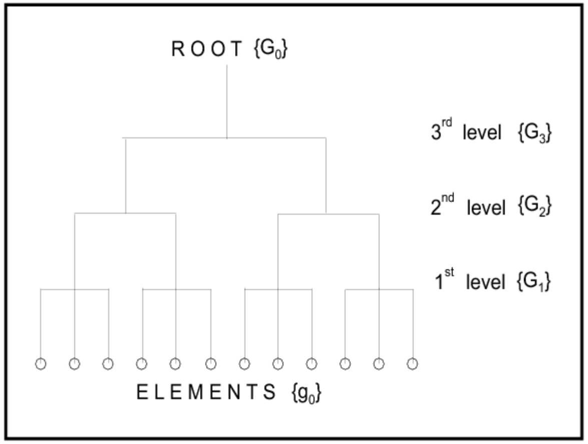
$m \sim r^{\alpha}$ scale invariance ($\alpha \sim 2$)

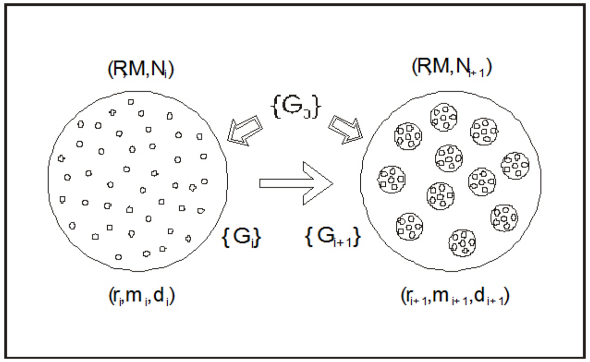
$$m_{i+1} = (2m_i^2 M)^{1/3}$$

$$r_{i+1} = (\sqrt{2} r_i^2 R)^{1/3}$$

(3) Network boundary conditions

e.g. ρ_c , H_0 or N_0 , m_0

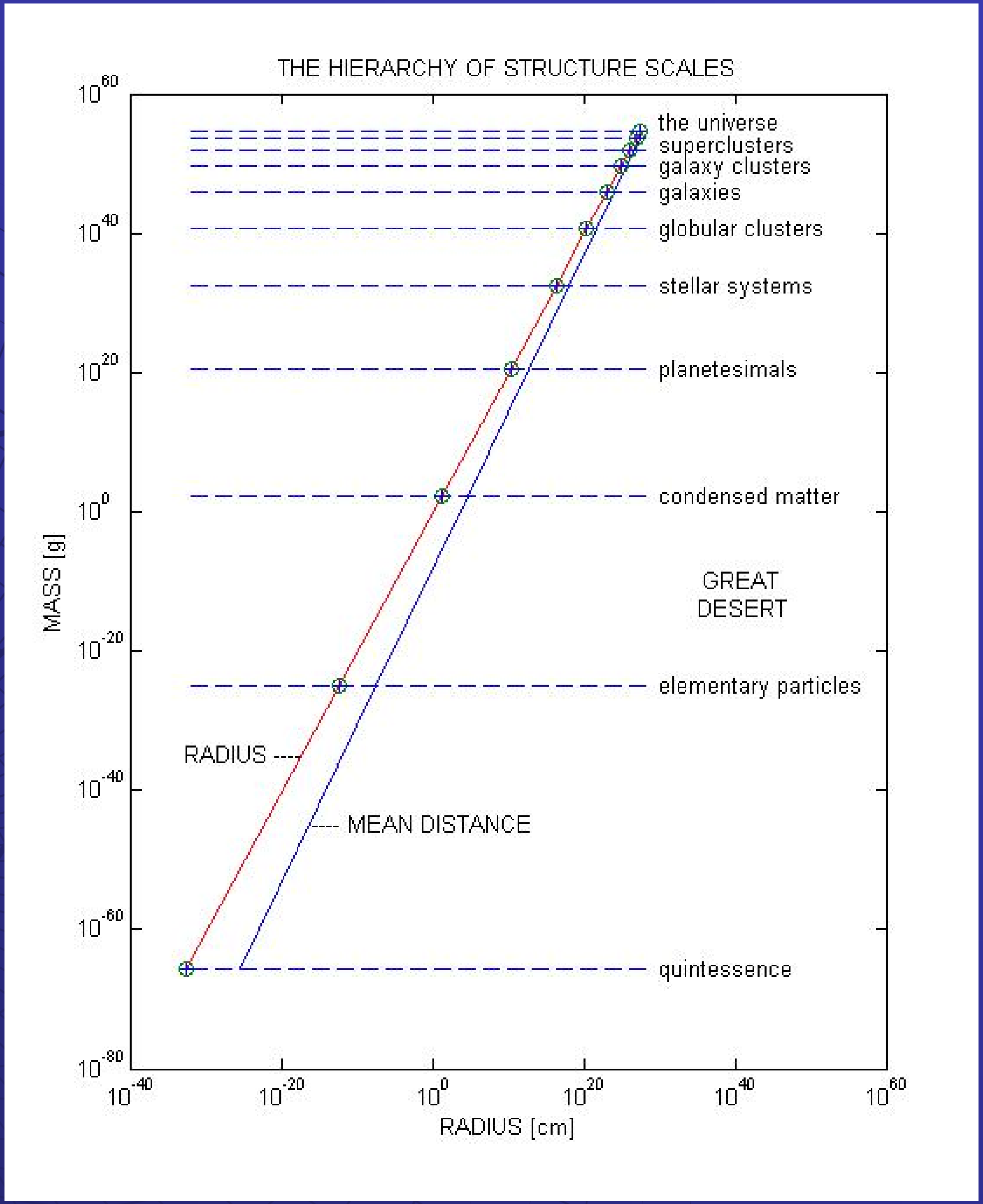




Radius – mass relation

hierarchical sequence of discrete cosmic structure scales

mimic time evolution by stepping up the hierachical tree towards larger cluster mergers



Sequence of structure scales

MASS	RADIUS	# MEMBERS	physical scale
8.5e+051	1.1e+026	3.0e+004	superclusters
3.5e+049	7.0e+024	7.3e+006	galaxy clusters
9.1e+045	1.1e+023	2.8e+010	galaxies
3.8e+040	2.3e+020	6.6e+015	globular clusters
3.3e+032	2.8e+016	7.6e+023	stellar systems
2.7e+020	1.9e+010	9.5e+035	planetesimals
1.9e+002	1.6e+001	1.3e+054	condensed matter
1.2e-025	4.1e-013	2.1e+081	particle scale
1.8e-066	1.6e-033	1.4e+122	quintessence

key role in evolutionary scenarios:

globular clusters – building blocks of galaxies
planetesimals – building blocks of stellar systems
condensed matter – nature appears highly complex
quintessence scalar field – m_I ~ 1e-66g

5 fundamental domains in NEXT statistics

K = - ∞   K = 0   K = 3/2   K = ∞

|________________|________|________________|

(1) ÷   (3)   2→ ÷   (4)   2→ ÷   (2)   2→ (1)

	(3)	(4)	(2)	
	$C < 0$	$C > 0$	$C > 0$	
	$p > 0$	$p < 0$	$p > 0$	
	$\rho > 0$	$\rho < 0$	$\rho > 0$	
BGS	self interaction (gravity) DM	highest correlations.. DE	correlations (electromagnetic) gen. thermostat.	BGS

C…heat capacity,   p…pressure,   ρ…density

(5) no dependence on entropic index … discrete structure scales

SUMMARY

nature appears nonlocal and nonlinear on all observable levels
requiring a generalization of standard BGS statistics

nonextensive theory accounts for long-range interactions
and correlations and is naturally subject to entropy bifurcation

BEYOND STANDARD BGS ENTROPY

the specific physical domains covered by the entropic index are manifest in

(1) astrophysical energy distributions and PDF's in plasma turbulence

(2) DM and plasma density distributions in gravitationlly clustered structures

(3) Repulsive DE landscape of negative equation of state

(4) Hierarchy of fundamental structure scales