COSMOLOGICAL k –ESSENCE CONDENSATION

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Dark energy/matter models

- ΛCDM- cosmological constant + dust (cold DM)
- QCDM - quintessence + dust

BEYOND the Standard Cosmology

- Quartessence –unified DE/DM

e.g., Chaplygin gas:

$$p = -\frac{A}{\rho}$$

the first definite model for a DE/DM unification

A. Kamenshchik, U. Moschella, V. Pasquier, PLB **511** (2001)
N.B., G.B. Tupper, R.D. Viollier, PLB **535** (2002)
J.C. Fabris, S.V.B. Goncalves, P.E. de Souza, GRG **34** (2002)

Chaplygin gas belongs to a wider class of models known as k-essence.

All models of DE/DM unification face the problem of $c_s \neq 0$. The linear perturbations of the scale smaller than the sonic horizon will be prevented from growing. Naive models turn out to be incompatible with observations. E. g., the Chaplygin gas model does not reproduce the mass power spectrum

H.B. Sandvik, M. Tegmark, M. Zaldarriaga, and I. Waga, PRD **69** (2004)

and the CMB

D. Carturan and F. Finelli, PRD **68** (2003);

L. Amendola et al, JCAP **07** (2003)

This unpleasant feature is a consequence of the linear perturbation theory and a non-vanishing comoving sonic horizon

$$d_s = \int dt \frac{c_s}{a} \qquad c_s^2 = \left.\frac{\partial p}{\partial \rho}\right|_S$$

The perturbation whose comoving size $R > d_s$ grow as

$$\delta = (\rho - \bar{\rho})/\bar{\rho} \sim a$$

As soon as $R<d_s$, δ undergoes dumped oscillations. For the Chaplygin gas $d_s \sim a^{7/2}/H_0$

It grows very fast and reaches Mpc scales already at $z \approx 20$

For large overdensities, $\delta > 1$, the perturbation theory cannot be trusted. Hence we must

- evolve the field equations nonperturbatively
- investigate whether a significant fraction of initial density perturbations collapses in gravitationally bound structures – the condensate

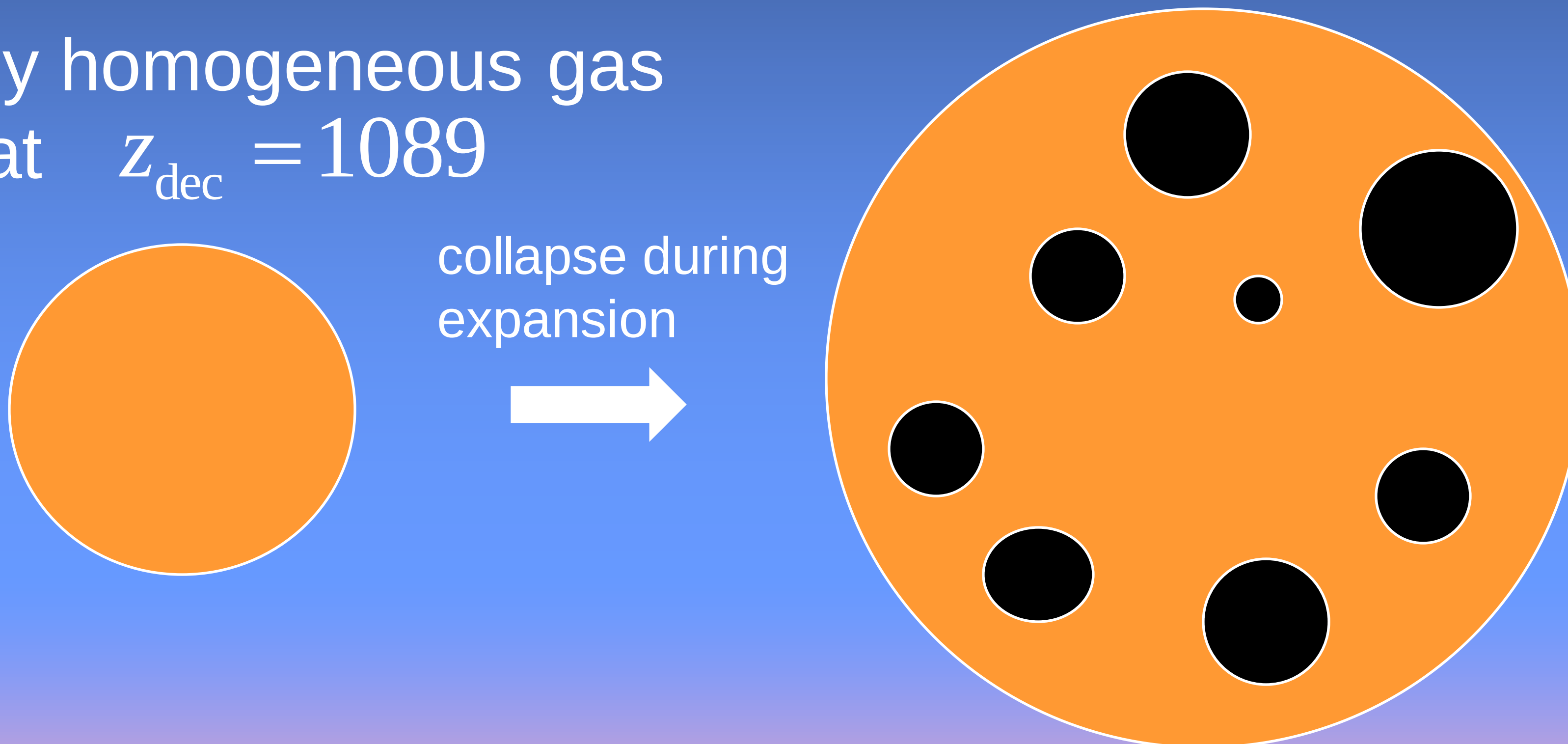
Two phase structure – mixture of CDM in the form of condensate and DE in the form of gas

gas + condensate

nearly homogeneous gas

at $z_{\mathrm{dec}} = 1089$

collapse during expansion



K-essentials

A general k-essence model is described by

$$S = \int d^4x \sqrt{-g} \left[-\frac{R}{16\pi G} + \mathcal{L}(\varphi, X) \right]$$

where $\mathcal{L} = V(\varphi)F(X) + U(\varphi)$ $\quad X = g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}$

We shall concentrate on the string theory inspired tachyon condensate* Lagrangian

$$\mathcal{L} = -V(\varphi)\sqrt{1-X}$$

*Should not be confused with ***gravitational condensate***

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In particular , we consider the potential

$$V(\varphi) = V_n \varphi^{2n} \qquad n = 0, 1, 2$$

n=0 gives the Dirac-Born-Infeld description of a D-brane - equivalent to the Chaplygin gas R. Jackiw, Lectures on Fluid Mechanics (Springer, 2002)

It may be shown that the model with *n≠0* effectively behaves as a ***variable*** Chaplygin gas with $p \sim -a^{6n} / \rho$. The much smaller sonic horizon $d_s \sim a^{(7/2+3n)} / H_0$ enhances condensate formation by 2 orders of magnitude.

N.B., G.B. Tupper, R.D. Viollier, PRD **80** (2009)

The constant V_n is adjusted so that the background evolution approximates the standard cosmology today with **Ω_Λ=0.73** and the equivalent matter content **Ω=1-Ω_Λ=0.27**

We solve the field equations with ***a*** starting from the **initial *a*$_{\mathbf{dec}}$** at decupling redshift ***z*$_{\mathbf{dec}}$=1089** for a chosen comoving size ***R***. The initial values $\bar{\rho}_{\text{in}}$, H_{in} for the background are chosen naturally as in the ΛCDM cosmology. For the initial inhomogeneity we take

$$\rho_{\text{in}} = \bar{\rho}_{\text{in}}(1+\delta_{\text{in}}); \quad \mathcal{H}_{\text{in}} = H_{\text{in}}\left(1-\delta_{\text{in}}/3\right)$$

Where $\delta_{\text{in}} = \delta_R(a_{\text{dec}})$ is a variable initial density contrast

N.B., G.B. Tupper, R.D. Viollier, PRD **80** (2009)

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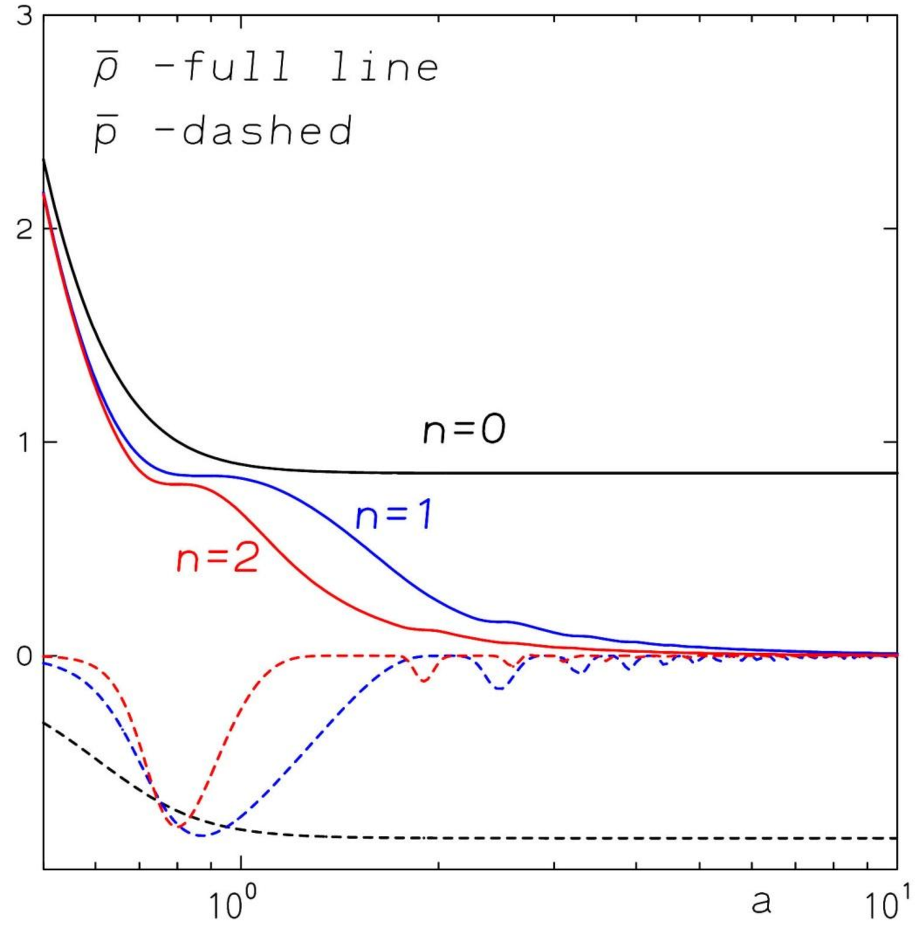
Evolution of the background

$$V(\varphi) = V_n \varphi^{2n}$$
$$n = 0, 1, 2$$



The background density and pressure versus *a* for the tachyon condensate model with the equivalent matter content *Ω=0.27.*

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Evolution of the background

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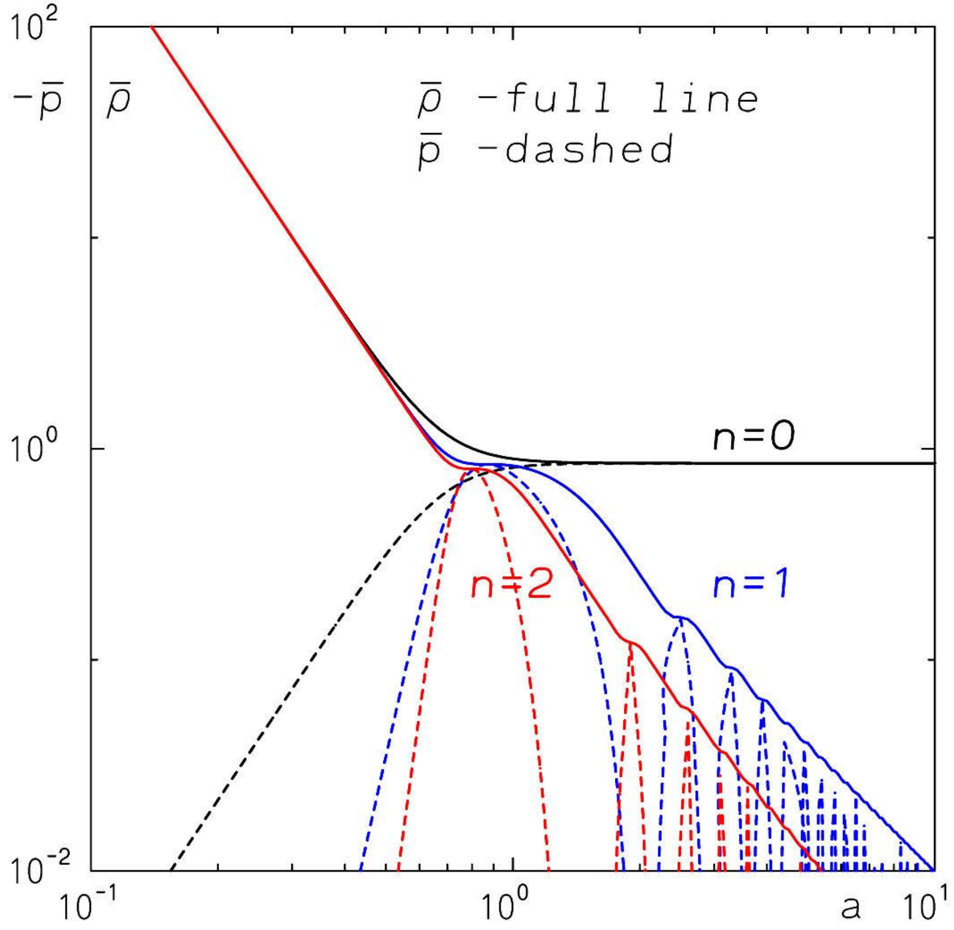


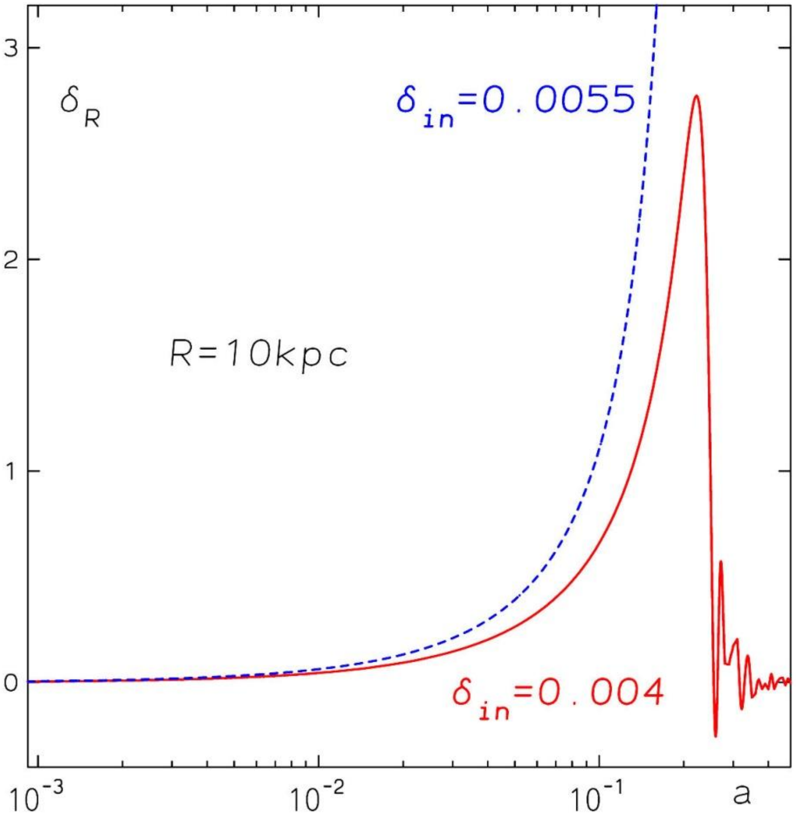
The background density and pressure versus *a* for the tachyon condensate model with the equivalent matter content *Ω=0.27.*

Evolution of the density contrast

Depending on the initial conditions, the density contrast

$$\delta = (\rho - \bar{\rho})/\bar{\rho}$$

described by the **nonlinear** evolution equations either grows as dust or undergoes damped oscillations



Density contrast of comoving size *R*=10 kpc as a function of *a* in the tachyon spherical model.

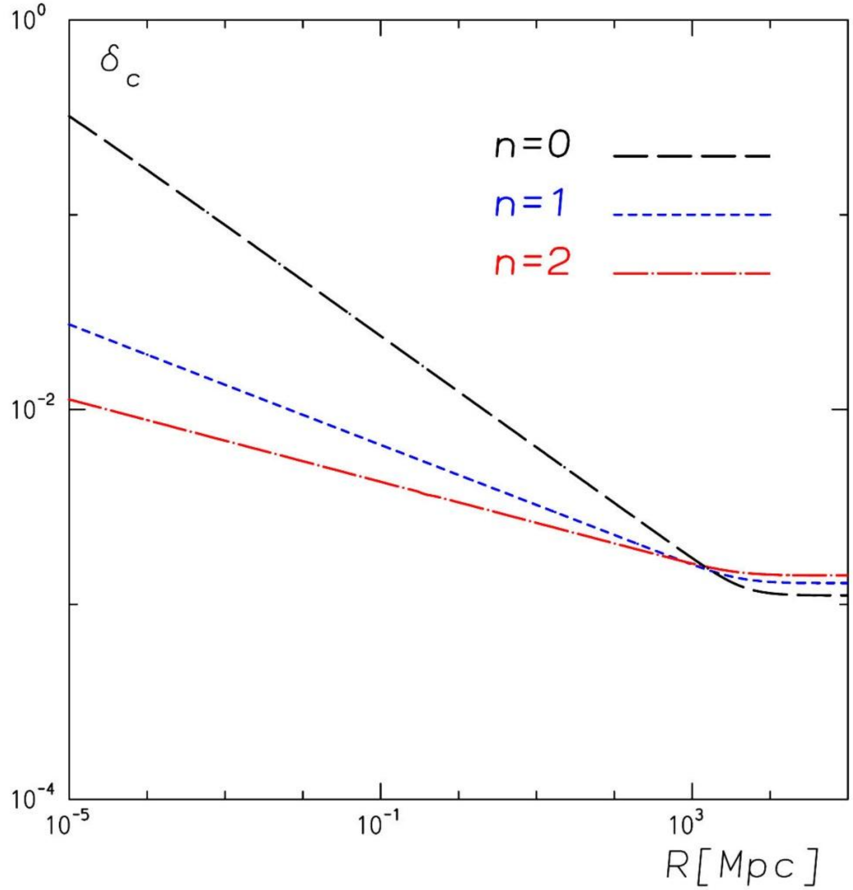
Perturbations with

$$\delta_{\rm in} \geq \delta_{\rm c}(R)$$

evolve into a nonlinear gravitational condensate that at low *z* behaves as dust. If

$$\delta_{\rm in} < \delta_{\rm c}(R)$$

the acoustic horizon can stop δ from growing; at low redshifts the perturbations behave as expected from linear theory



The threshold δ_c that separates the condensate regime from the damped oscillating regime as a function of comoving size *R*

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The Condensate Fraction

The crucial question is what fraction of the tachyon gas goes into condensate. If the fraction is sufficiently large the CMB and the mass power spectrum could be reproduced

N.B., R.J. Lindebaum, G.B. Tupper, R.D. Viollier, in conf proc. of XV Rencontre de Blois, astro-ph/0310181

We apply the Press-Schechter formalism. Assuming that $\delta_R\,(a_{\mathrm{dec}})$ is a Gaussian random field with dispersion σ(R), the condensate fraction is

$$F(R) = 2\int_{\delta_c}^{\infty} \frac{d\delta}{\sqrt{2\pi\sigma}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right) = \mathrm{erfc}\left(\frac{\delta_c(R)}{2\sigma(R)}\right)$$

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The dispersion

$$\sigma(R)^2 = \int_0^\infty \frac{dk}{k} \exp\left(-k^2 R^2\right) \Delta^2(k, a_{\mathrm{dec}})$$

is calculated in

N.B., R. Lindebaum, G.B. Tupper, R.D. Viollier, JCAP **0411** (2003)

using the variance of the concordance model

$$\Delta^2(k,a) = \mathrm{const}\left(\frac{k}{aH}\right)^4 T(k)^2 \left(\frac{k}{7.5H_0}\right)^{n_s - 1}$$

with the spectral index n_s=1.02 and the transfer function $T(k)$ parameterized as in

J.M. Bardeen *et al*, Astrophys. J. **187** (1986)

with Ω_B=0.04

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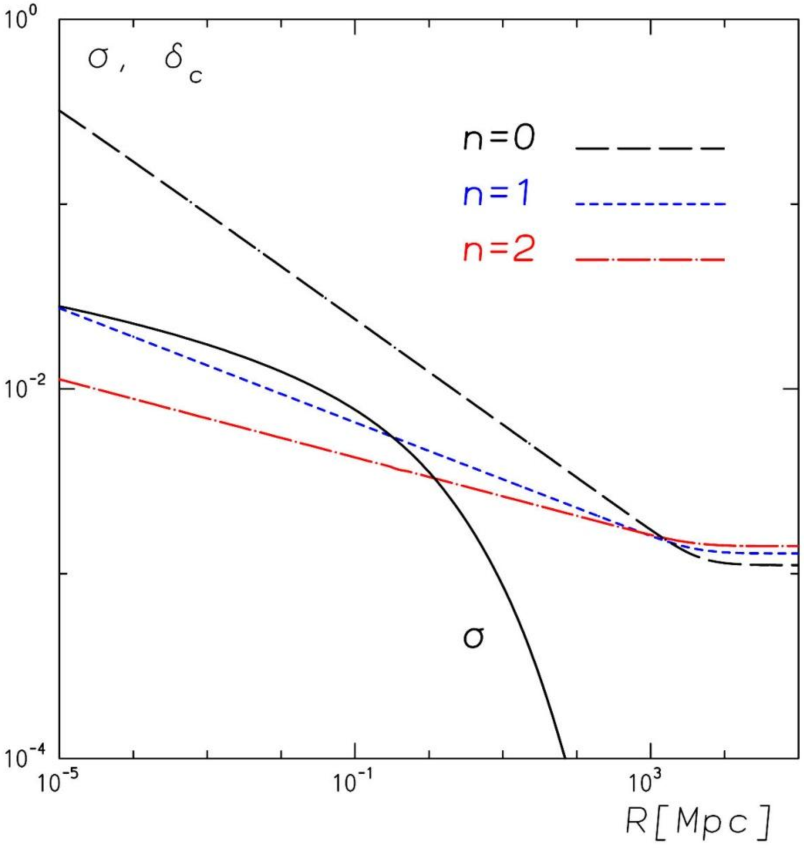
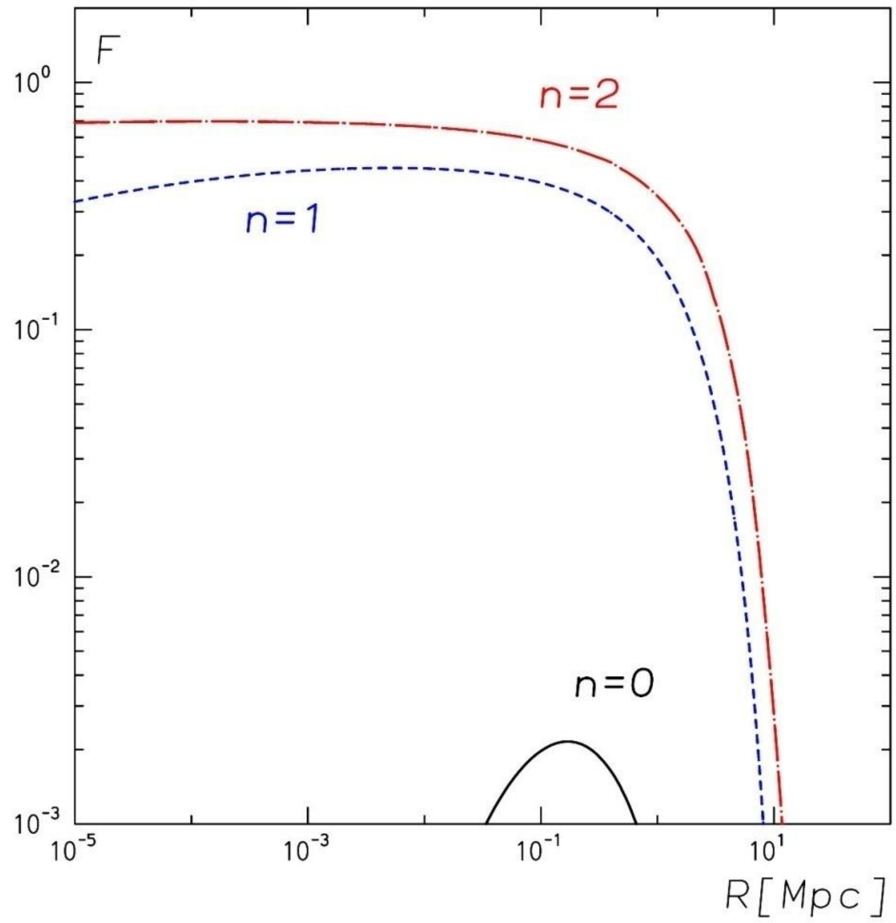
The dispersion σ calculated using Δ^2 of the concordance model.
The parameters are fixed by fitting the 2dFGRS power spectrum

W.J. Percival *et al*, MNRAS **327** (2001)



The dispersion σ in comparison with the threshold δ_c



Fraction of the tachyon gas in collapsed objects using $\delta_c(R)$ and $\sigma(R)$ from the previous figure

Hydrodynamic Description

$$\mathcal{L} = -V(\varphi)\sqrt{1 - X}$$

$$X = g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu}$$

For X>0 (generally OK in cosmological setting) the energy-momentum tensor is of the perfect fluid form

$$T_{\mu\nu} = 2\frac{\partial \mathcal{L}}{\partial X}\varphi_{,\mu}\varphi_{,\nu} - \mathcal{L}\, g_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}$$

with $u_\mu = \mathrm{sgn}(\varphi_{,0})\dfrac{\varphi_{,\mu}}{\sqrt{X}}$ $\quad p = \mathcal{L}; \quad \rho = 2X\dfrac{\partial \mathcal{L}}{\partial X} - \mathcal{L}$

The evolution is described by the Raychaudhuri equation for the velocity congruence combined with the Einstein and Euler equations

$$3\dot{\mathcal{H}} + 3\mathcal{H}^2 + \sigma_{\mu\nu}\sigma^{\mu\nu} + 4\pi G(\rho + 3p) = \left(\frac{c_s^2 h^{\mu\nu} \rho_{,\nu}}{\rho + p} \right)_{,\mu}$$

where

$$h_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$$

$$\mathcal{H} = 1/3 u^\mu{}_{,\mu}$$ local Hubble parameter

$$\sigma_{\mu\nu} = h_\mu^\alpha h_\nu^\beta u_{(\alpha;\beta)} - \mathcal{H} h_{\mu\nu}$$ shear tensor

Assuming spherical symmetry, this equation may be solved numerically (R. Lindebaum)

The Spherical Model

In spherically symmetric collapse the space time is conveniently described by the metric

$$ds^2 = N(t,r)^2 dt^2 - b(t,r)^2 \left(dr^2 + r^2 f(t,r) d\Omega^2 \right)$$

with FRW spatially flat asymptotic geometry so that $N \to 1, \quad f \to 1, \quad b \to a(t)$ as $r \to \infty$

b is the local expansion scale and the local Hubble parameter is given by

$$\mathcal{H} = \frac{1}{N} \left(\frac{\dot{b}}{b} + \frac{1}{3} \frac{\dot{f}}{f} \right)$$

Local approximation: the density contrast δ is assumed to be of fixed Gaussian shape with comoving size *R,* so that

$$\rho(t,r) = \bar{\rho}(t)[1+\delta_R(t)e^{-r^2/(2R^2)}]$$

and the spatial derivatives are calculated at r=0. The functions $N, b, \mathcal{H}$ near r=0 behave as

$$N(t,r) = N(t)[1+O(r^2)]$$

Due to the deffinition of u_μ, the field φ in comoving coordinates is a function of time only. In this way we obtain a set of 6 *ordinary* differential equations for $\rho, \bar{\rho}, H, \varphi, b, \mathcal{H}$

$$\dot{\varphi}^2 = 1 - \frac{V(\varphi)^2}{\bar{\rho}^2}$$

$$\dot{\bar{\rho}} + 3H(\bar{\rho} + \bar{p}) = 0$$

$$\dot{H} + H^2 + \frac{4\pi G}{3}(\bar{\rho} + 3\bar{p}) = 0$$

background

$$N = \left(\frac{1 - V(\varphi)^2 / \rho^2}{1 - V(\varphi)^2 / \bar{\rho}^2} \right)^{1/2}$$

$$\dot{b} = N b \mathcal{H}$$

$$\dot{\rho} + 3N\mathcal{H}(\rho + p) = 0$$

$$\frac{1}{N}\dot{\mathcal{H}} + \mathcal{H}^2 + \frac{4\pi G}{3}(\rho + 3p) = \frac{c_s^2(\rho - \bar{\rho})}{b^2 R^2 (\rho + p)}$$

Spherical inhomogeneity

The constant V_n is adjusted so that the background evolution approximates the Standard cosmology with **Ω_Λ=0.73**.

We solve the field equations with ***a*** starting from the **initial *a*$_{\mathbf{dec}}$** at decupling redshift ***z*$_{\mathbf{dec}}$=1089** for a chosen comoving size ***R***. The initial values are

$$\bar{\rho}_{\text{in}} = \rho_0 \frac{\Omega}{a_{\text{dec}}^3}; \quad H_{\text{in}} = H_0 \sqrt{\frac{\Omega}{a_{\text{dec}}^3}}; \quad \varphi_{\text{in}} = \frac{2}{3H_{\text{in}}}$$

$$\rho_{\text{in}} = \bar{\rho}_{\text{in}} (1+\delta_{\text{in}}); \quad \mathcal{H}_{\text{in}} = H_{\text{in}} \left(1-\delta_{\text{in}}/3\right)$$

$\Omega = 1 - \Omega_\Lambda = 0.27$ is the equivalent matter content

$\delta_{\text{in}} = \delta_R(a_{\text{dec}})$ is a variable initial density contrast

N.B., G.B. Tupper, R.D. Viollier, PRD **80** (2009)

Summary and Conclusions

- We have demonstrated that a significant fraction of the tachyon-like k-essence fluid with the potential $V \sim \varphi^4$ collapses into condensate objects that play the role of DM
- These results were obtained in a relativistic framework for nonlinear evolution that includes the effects of the pressure gradient and the acoustic horizon
- The tachyon k-essence unification remains to be tested against large-scale structure and CMB observations