BEYOND THE STANDARD MODELS OF PARTICLE PHYSICS, COSMOLOGY AND ASTROPHYSICS

Cape Town, South Africa, 1-6 February 2010

Detecting of Relic Neutrinos and Measuring Fundametal Properties of Neutrinos with Atomic Nuclei

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A. Barabash (ITEP Moscow) …

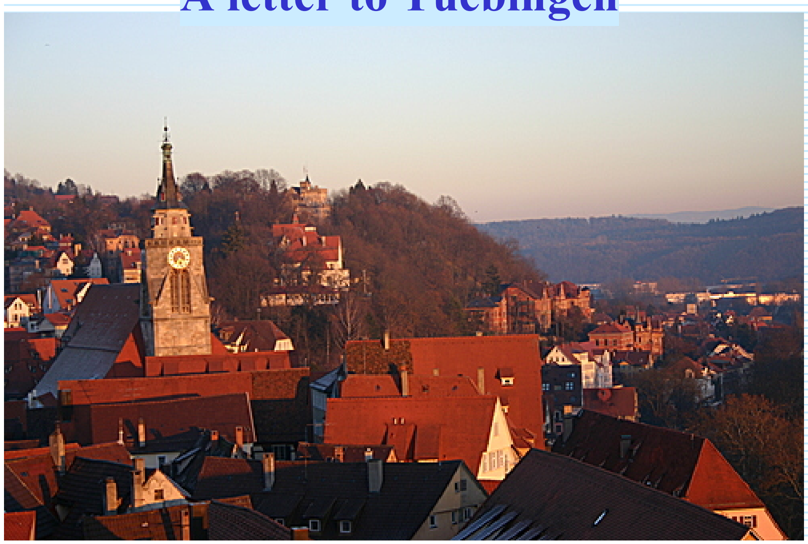
OUTLINE

- Introduction
- Absolute neutrino mass scale and single β-decay
- The 0νββ–decay NMEs
- Oscillations of atoms (DEC)
- (Partly)bosonic neutrino and 2νββ-decay
- Conclusion and outlook

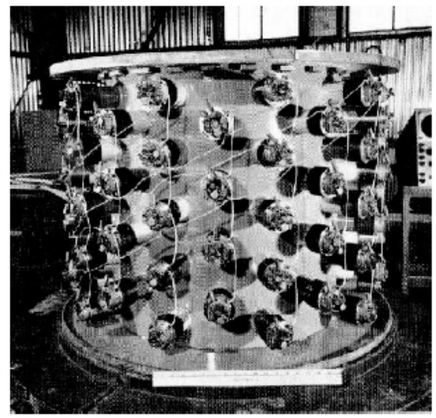
Pauli proposes existence of "neutron" (with spin ½ and mass not more than 0.01 mass of proton) in nucleus. β-decay is then a three body decay with continues distribution of energy among constituents.

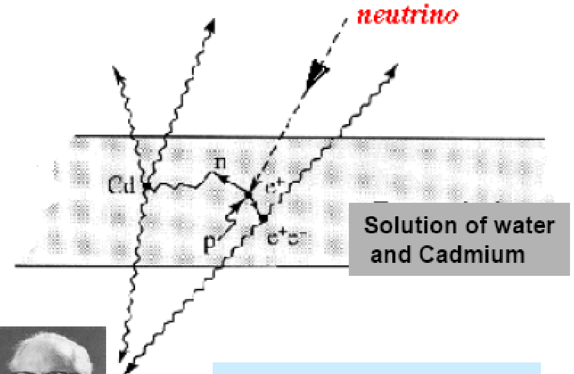
I have done a terrible thing
I invented a particle that cannot be detected
W. Pauli

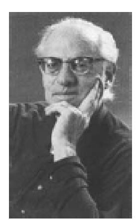
4 December 1930
A letter to Tuebingen



Detector at Savannah River
Nuclear reactor (1956)







$$\bar{\nu} + p \rightarrow n + e^{+}$$

Reines: 1995 Nobel Prize

3 events per hour

We are happy to inform you (Pauli) that we have definitely detected ν
Reines & Cowan

Signals due to:

i) e^{+} annihilation,

ii) n-capture

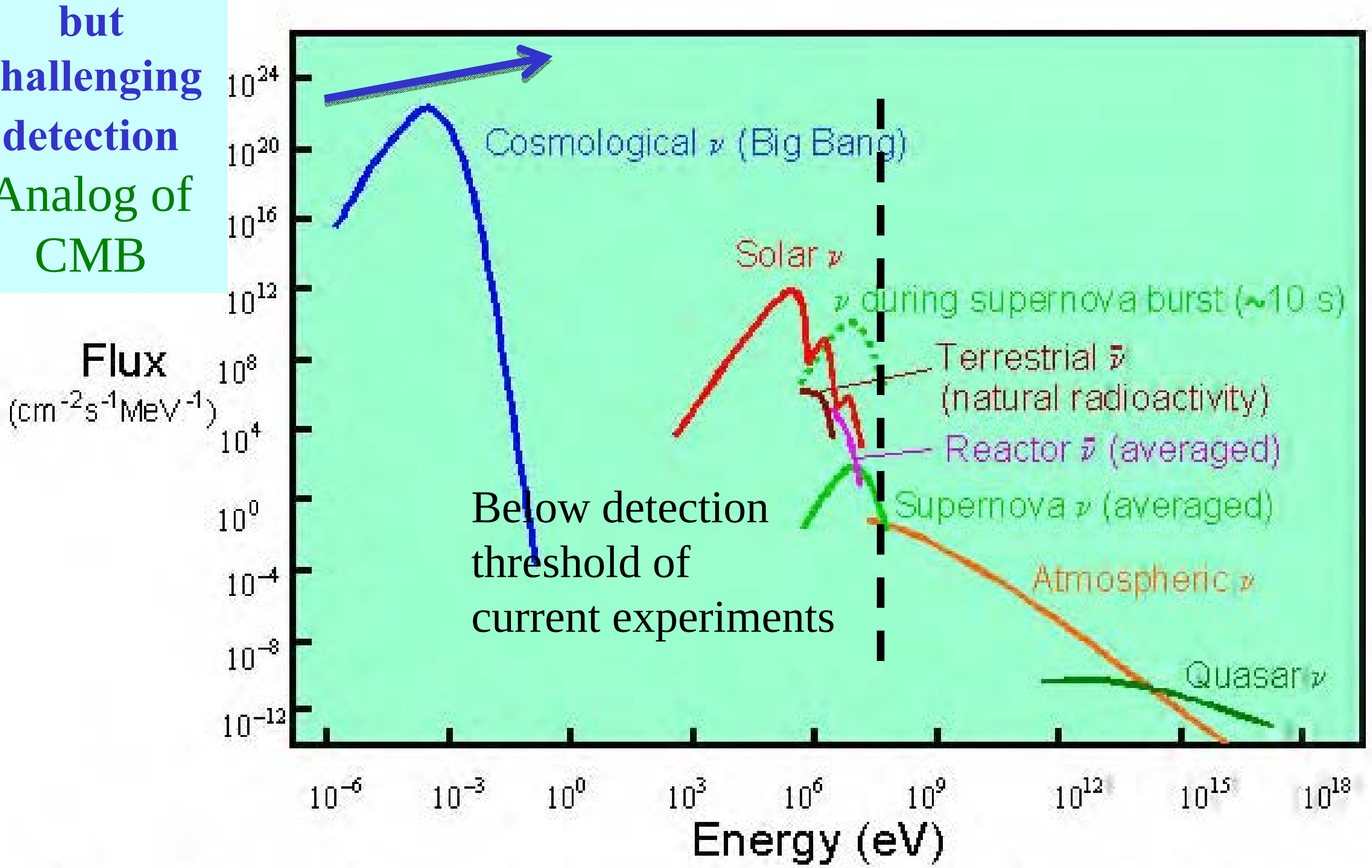
$$\sigma = (1.1\pm0.3)\ 10^{-43}\ cm^{2}$$

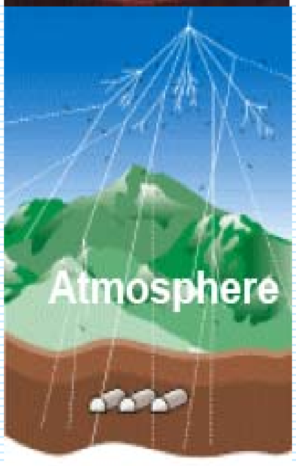
in agreement with Fermi theory of β-decay

Sources of neutrinos

The Sun is the most intense detected source with a flux on Earth of 6 10^{10} ν/cm^2s

Abundant but challenging detection
Analog of CMB





Flux on Earth of neutrinos from different sources as a function of energy

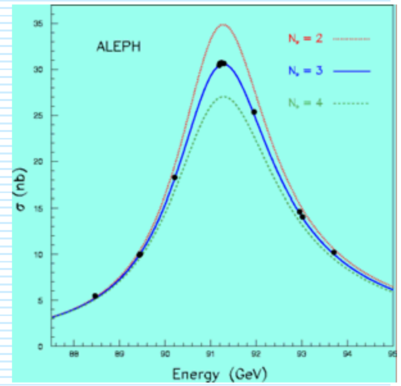
D. Vignaud and M. Spiro, Nucl. Phys.A 654 (1999) 350

Fundamental properties of neutrinos

Like most people, physicists enjoy a good mystery. When you start investigating a mystery you rarely know where it is going

After 54 years we know

- **3 families of light (V-A) neutrinos:** ν_e, ν_μ, ν_τ
- **ν are massive:** we know mass squared differences
- **relation between flavor states and mass states (neutrino mixing) only partially known**



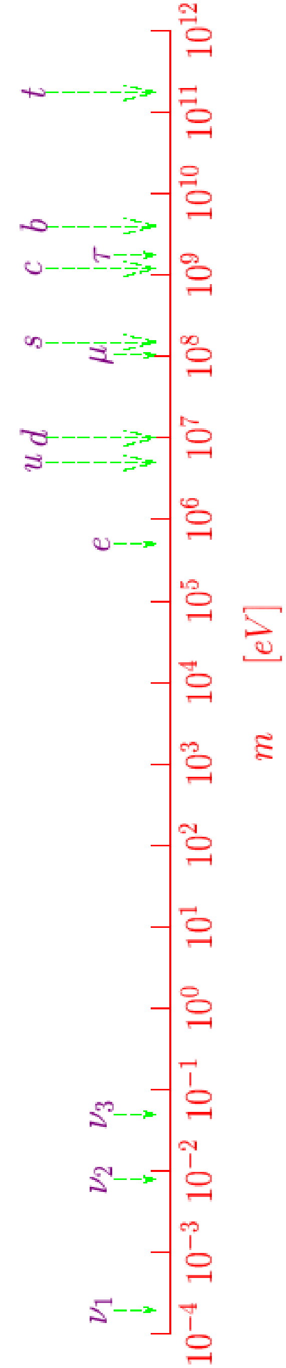
Claim for evidence of the $0\nu\beta\beta$-decay

H.V. Klapdor-Kleingrothaus et al.,NIM A 522, 371 (2004); PLB 586, 198 (2004)

- **Absolute ν mass scale from the $0\nu\beta\beta$-decay. (cosmology, ^{3}H, ^{187}Rh ?)**
- **ν's are their own antiparticles – Majorana.**

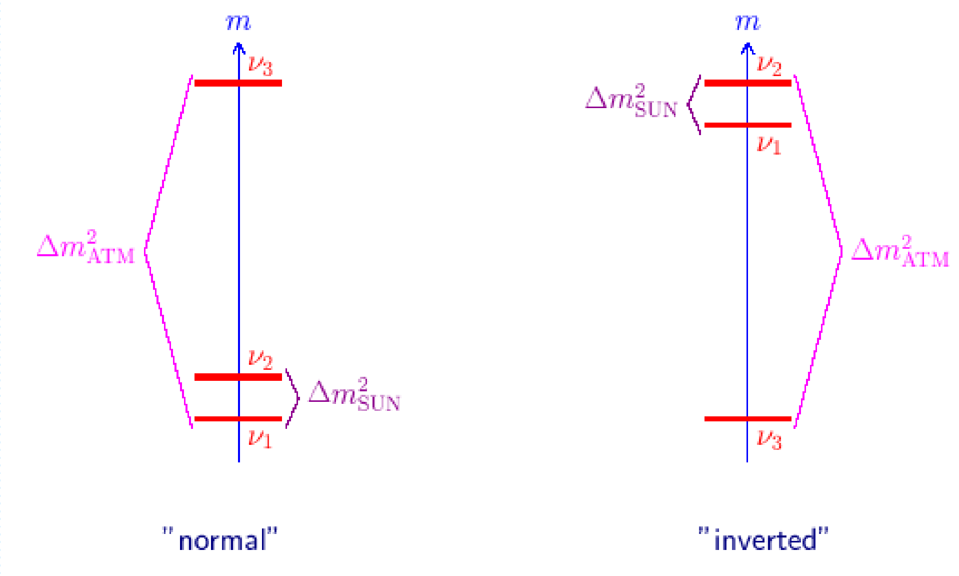
No answer yet

- **Is there a CP violation in ν sector? (leptogenesis)**
- **Are neutrinos stable?**
- **What is the magnetic moment of ν?**
- **Sterile neutrinos?**
- **Statistical properties of ν? Fermionic or partly bosonic?**



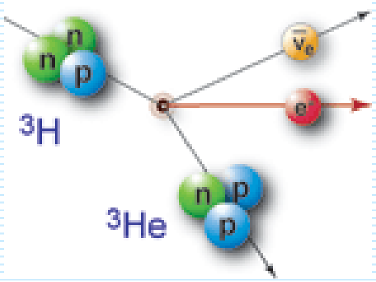
Absolute ν mass scale and β-decay of ^3H and ^{187}Re

ν-oscillations



$$m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 \; m_i^2}$$

Tritium beta decay: $^3H \rightarrow {}^3He + e^- + \bar{\nu}_e$

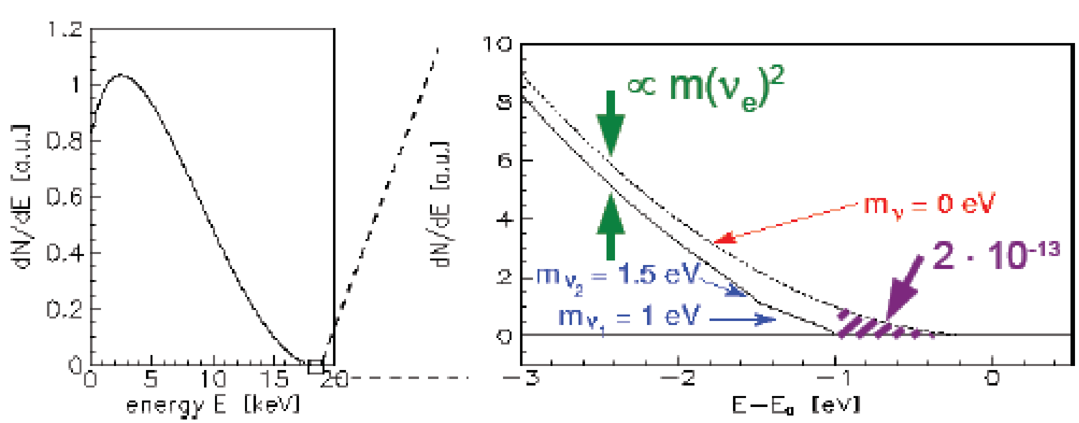


$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

1934 – Fermi pointed out that shape of electron spectrum in β–decay near the endpoint is sensitive to neutrino mass

First measured by Hanna and Pontecorvo with estimation $m_\nu \sim 1$ keV [Phys. Rev. 75, 983 (1940)]

$Q = M_H - M_{He} - m_e = 1858$ keV



Troitsk

$m_\nu^2 = -2.3 \pm 2.5 \pm 2.0\ \text{eV}^2$

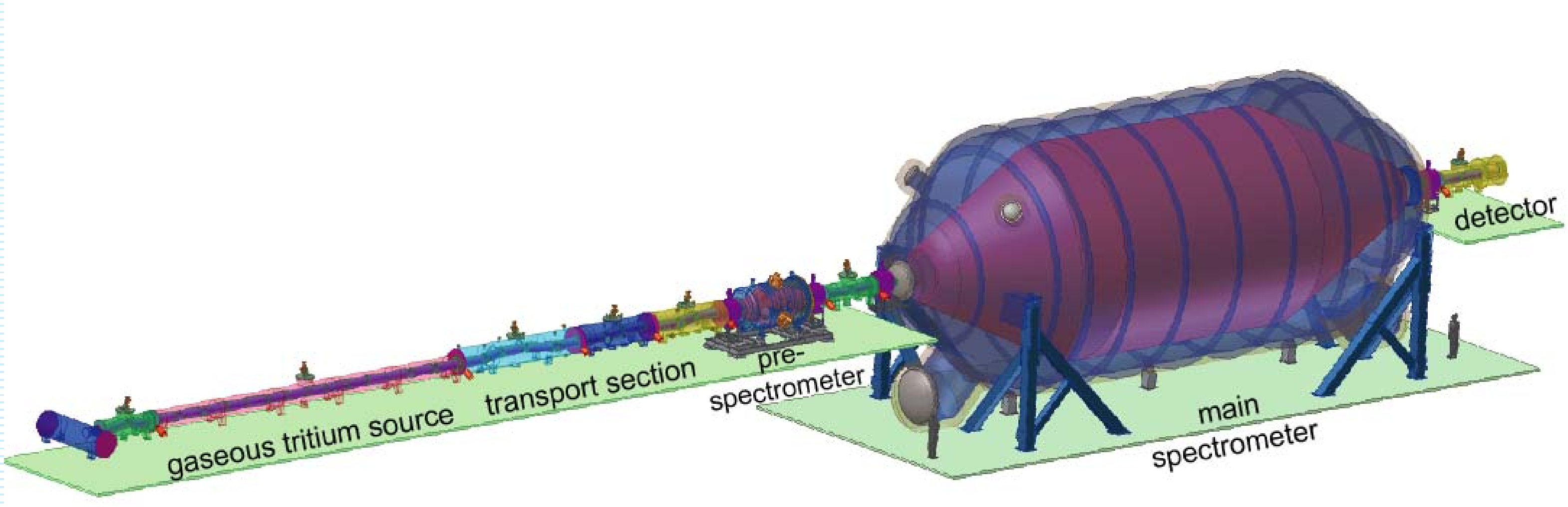
$m_\nu \leq 2.2$ eV (95% CL.)

Mainz

$m_\nu^2 = -1.2 \pm 2.2 \pm 2.1\ \text{eV}^2$

$m_\nu \leq 2.2$ eV (95% CL.)

Karlsruhe TRItium Neutrino experiment (KATRIN)



Evidence for neutrino mass signal
KATRIN discovery potential:

No neutrino mass signal
KATRIN sensitivity

$$\mathbf{m_\beta = 0.35\ eV\ (5\sigma)}$$
$$\mathbf{m_\beta = 0.30\ eV\ (3\sigma)}$$

$$m_\beta = \sqrt{\sum_{i=1}^{3} |U_{ei}|^2 m_i^2} < 0.2\ eV \qquad m_\beta \approx m_1$$

Relativistic approach to ^{3}H decay

Standard approach
- **non-relativistic nuclear w.f.**
- **nuclear recoil neglected**
- **phase space analysis**

$$E_e^{\max} = M_i - M_f - m_\nu$$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E)\, pE\, (Q-T) \sqrt{(Q-T)^2 - m_{\nu_e}^2}$$

Relativistic EPT approach
- **Analogy with n-decay**
  (^{3}H,^{3}He) ↔ (n,p)
- **nuclear recoil of 3.4 eV by E_e^{max}**
- **relevant only phase space**

$$E_e^{\max} = \frac{1}{2M_f}\left[M_i^2 + m_e^2 - \left(M_f^2 - m_\nu^2\right)\right]$$

$$\begin{aligned}
\frac{d\Gamma}{dE_e} &= \frac{1}{(\pi)^3}(G_F\cos\theta_c)^2 F(Z,E_e)p_e \\
&\times \frac{M_i^2}{(m_{12})^2}\sqrt{y\left(y+2m_\nu\frac{M_f}{M_i}\right)} \\
&\times \Bigg[(g_V+g_A)^2 y\left(y+m_\nu\frac{M_f}{M_i}\right)\frac{M_i^2(E_e^2-m_e^2)}{3(m_{12})^4} \\
&\quad (g_V+g_A)^2 (y+m_\nu\frac{M_f+m_\nu}{M_i})\frac{(M_iE_e-m_e^2)}{m_{12}^2} \\
&\quad \times (y+M_f\frac{M_f+m_\nu}{M_i})\frac{(M_i^2-M_iE_e)}{m_{12}^2} \\
&\quad - (g_V^2-g_A^2)M_f\left(y+m_\nu\frac{(M_f+M_\nu)}{M_i}\right) \\
&\quad \times \frac{(M_iE_e-m_e^2)}{(m_{12})^2} \\
&\quad + (g_V-g_A)^2 E_e\left(y+m_\nu\frac{M_f}{M_i}\right)\Bigg]
\end{aligned}$$

$$\begin{aligned}
y &= E_e^{max} - E_e \\
(m_{12})^2 &= M_i^2 - 2M_iE_e + m_e^2
\end{aligned}$$

Numerics:
Practically the same dependence of Kurie function on m_ν for $E_e \approx E_e^{max}$

F.Š., R. Dvornický, A. Faessler, PRC 77 (2008) 055502

Rhenium β-decay

$$^{187}Re \rightarrow {}^{187}Os + e^- + \tilde{\nu}_e$$

- **Beta emitter of g.s.→g.s. transition with lowest known Q value (2.47 keV)**
- **Relative high half-live ($T_{1/2}$=4.35 x 10^{10} y) ~ age of the Universe**
- **Natural abundance 63%**

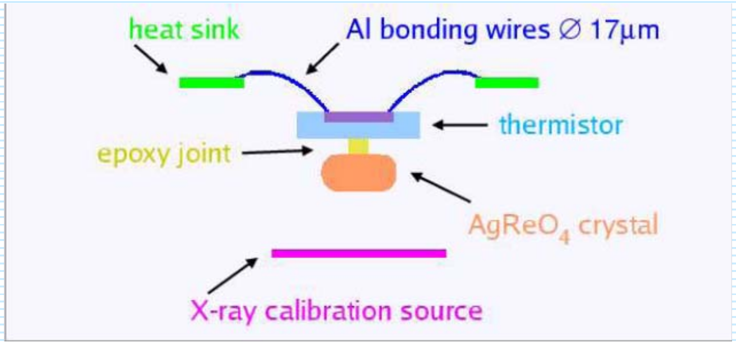
first unique forbidden β-decay ⇒ $5/2^+ \rightarrow 1/2^- \Rightarrow \Delta J^\pi = 2^-$

MIBETA ($AgReO_4$, 10*(250-350) mg Milano/Como)
MANU (Re metalic crystals, 1.5 mg, Genova)

m_ν^2 = -141±211±90 eV^2
m_ν = 15.6 eV (90% c.l.)

The entire energy is measured in the detector except the neutrino including the molecular & atomic excitations

M.Sisti et al., NIMA 520 (2004) 125



Microcalorimeter Arrays for a Rhenium Experiment (MARE)

MARE II: 5000 – 50 000 detectors (MIBETA 10)
Expected sensitivity m_ν = 0.2 eV

Spectrum of emitted electrons in rhenium β-decay

$$\frac{d\Gamma}{dE} = \frac{G_F^2 V_{ud}^2}{2\pi^3} |M|^2 \, pE(E_0 - E)\sqrt{(E_0 - E)^2 - m_\nu^2} \, \frac{1}{3} R^2 \left(p^2 F_1(Z,E) + k^2 F_0(Z,E) \right)$$

$$k = \sqrt{(E_0 - E)^2 - m_\nu^2}$$

Electron $p_{3/2}$ decay channel clearly dominates

$$\Gamma_S / \Gamma_P = 1.011 \times 10^{-4}$$

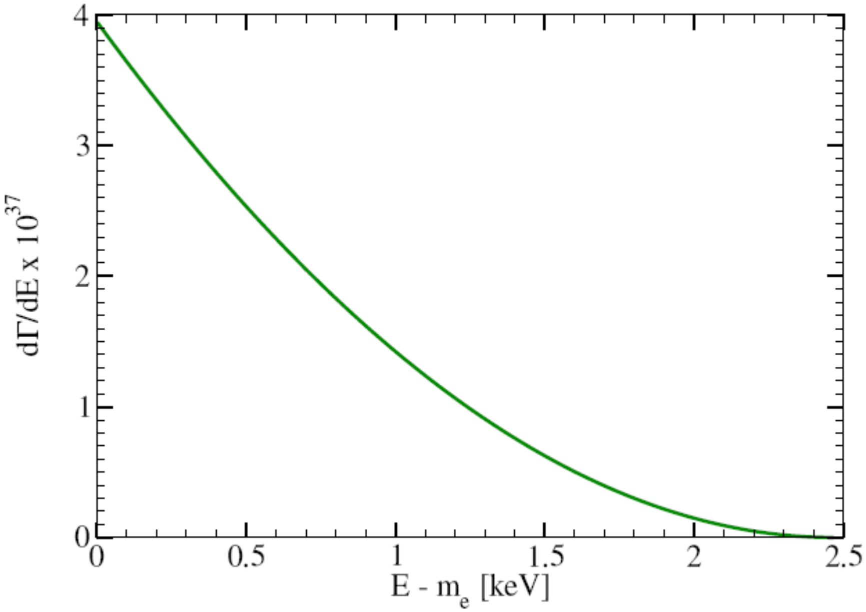
In agreement with
Arnaboldi et al.: PRL 96, 042503 (2006)

Electron in the $p_{3/2}$ state

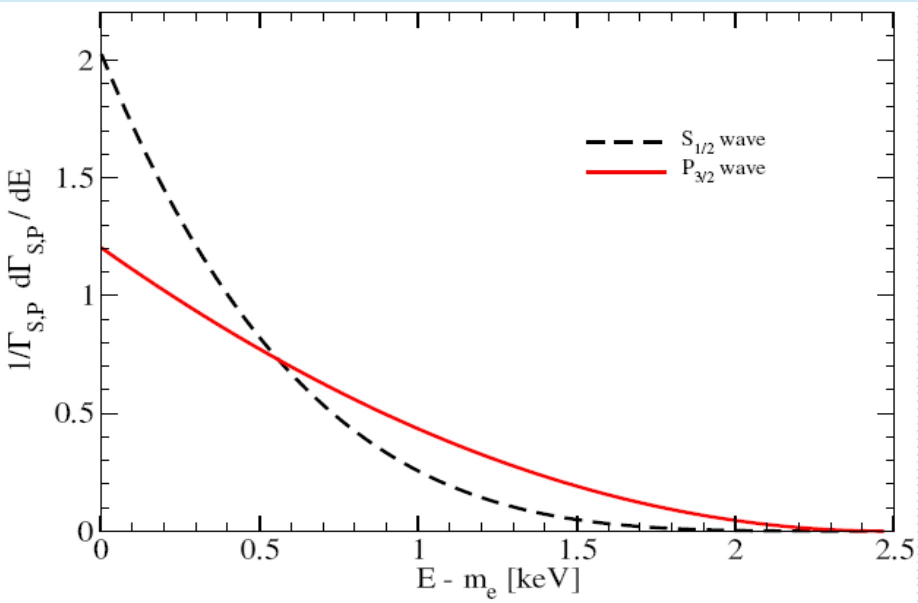
$$p^{\max} \cong 50 keV$$

Electron in the $s_{1/2}$ state

$$k^{\max} = 2.47 keV$$



lor S



Kurie plots for rhenium (MARE) and tritium (KATRIN) β-decay

Rhenium

$$B_{\mathrm{Re}} = \frac{G_F V_{ud}}{\sqrt{2\pi^3}} \frac{g_A}{\sqrt{2J_i+1}} \left| <^{187}Os \,||\, \sqrt{\frac{4\pi}{3}} \sum_n \tau_n^+ \frac{r_n}{R} \{\sigma_1 \otimes Y_1\}_2 \,||^{187} Re > \right|$$

$$\times \sqrt{\frac{1}{3} R^2 p^2 \frac{F_1(Z,E)}{F_0(Z,E)}}$$

$$K(E_e)/B_{\mathrm{Re}} \cong (E_0 - E_e) \sqrt[4]{1 - \frac{m_\nu^2}{(E_0 - E_e)^2}}$$

Tritium

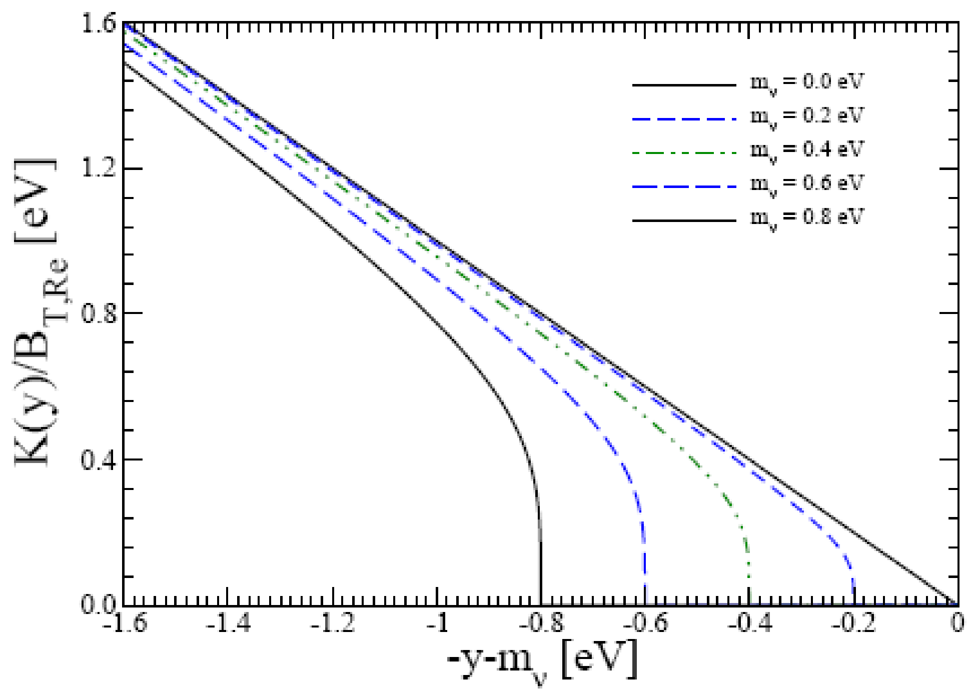
$$B_T = \frac{G_F V_{ud}}{\sqrt{2\pi^3}} \sqrt{g_V^2 + 3g_A^2}$$

$$K(y)/B_T = \left(\sqrt{y(y+2m_\nu)}\,(y+m_\nu)\right)^{1/2}$$

$y = E_e^{max} - E_e$

Properly normalized Kurie functions are practically the same by the endpoint !

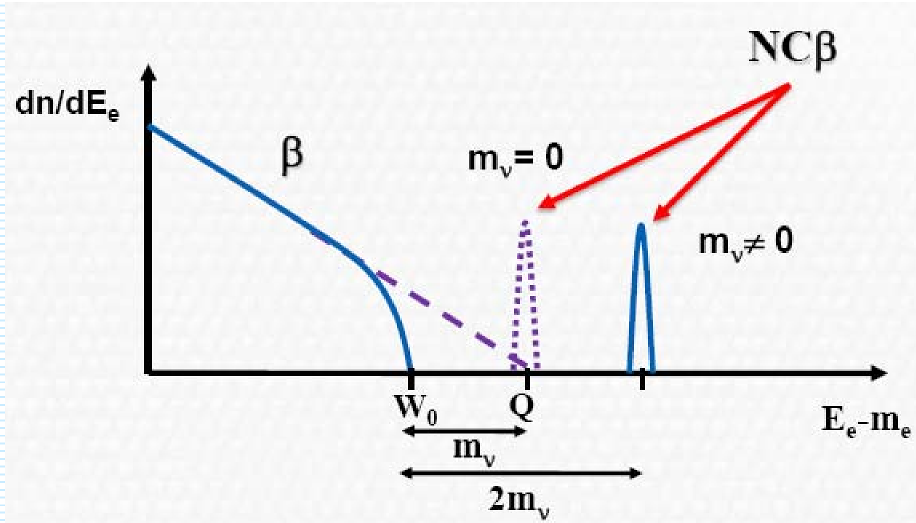
$$\mathbf{K(E)/B_{Re} \cong K(y)/B_T}$$



Relic neutrinos

The neutrino capture via the β–decaying nucleus is a unique tool to detect cosmological neutrinos

$$\nu + (A,Z) \rightarrow (A,Z+1) + e^-$$



The density of relic ν: <η>=56 cm^{-3}

Temperature

$$T_\nu^{\;0} = \left(\frac{4}{11}\right)^{1/3} T_\gamma^{\;0} \approx (1.945 \pm 0.001) K \rightarrow \mathrm{k_B} T_\nu \approx (1.676 \pm 0.001) \times 10^{-4}\, eV$$

$$T_\gamma^{\;0} = (2.725 \pm 0.001) K = (2.348 \pm 0.001) \times 10^{-4}\, eV$$

Mean momentum

$$\langle p_\nu^0 \rangle = \frac{7}{2} \frac{\zeta(4)}{\zeta(3)} T_\nu^0 \approx 3.151 T_\nu^0 \approx 5.314 \times 10^{-4}\, eV$$

Gravitational clustering of relic neutrinos

- Neutrinos of CνB are non-relativistic and weakly-clustered
- If they are heavy enough, such that their velocities become less than the escape velocity of a massive object, the RNs fall into the potencial wells of the latter – and are clustered today
- Massive neutrinos, $m_\nu \sim 1$ eV, will be gravitationally clustered on the scale of ~Mpc ($\sim 3\times 10^{19}$km), that is on the scale of galaxy clusters
- Overdensities of the order of 10^3-10^4

R. Lazauskas, P. Vogel, C. Volpe, J. Phys. G: Nucl. Part. Phys. 35 (2008)

Detection of relic neutrinos by KATRIN experiment

$$\nu + {}^3H((1/2)^+) \rightarrow {}^3He((1/2)^+) + e^-$$

Assuming $M_F=1$, $M_{GT}=\sqrt{3}$ and $\eta_\nu=<\eta_\nu>$ the capture rate

$$\Gamma^\nu({}^3H) = \frac{1}{\pi} G_\beta^2\, F_0(2,p)\, p\, p_0\, \left(|M_F|^2 + g_A^2 |M_{GT}|^2\right) \frac{\eta_\nu}{<\eta_\nu>} <\eta_\nu>$$

$$\Gamma^\nu({}^3H) = 4.2\ \ 10^{-25}\ \ y^{-1}$$

Ratio of capture and decay rates

A. G. Cocco, G. Mangano, M. Messina

6.6 10^{-24}

$T_{1/2}$=12.32 y ⇒

$$\frac{\Gamma^\nu({}^3H)}{\Gamma^\beta({}^3H)} = 7.5\ 10^{-24}$$

KATRIN will use ~50 μg of ^3H

$$N^\nu_{capt}(KATRIN) \approx 4.2\ 10^{-6}\ \frac{\eta_\nu}{<\eta_\nu>}\ y^{-1}$$

Even considering effect of clustering of ν, $\eta_\nu/<\eta_\nu>$ ~ 10^3-10^4 :

N^ν_{capt}(KATRIN) < 1 y^{-1}

Detection of relic neutrinos by MARE experiment

$$\nu + {}^{187}Re((5/2)^+) \rightarrow {}^{187}Os((1/2)^-) + e^-$$

The capture rate

$$\Gamma^\nu({}^{187}Re) = \frac{1}{\pi} G_\beta\, F_1(76,p)\, \frac{1}{3}\, (p\, R)^2\, \mathscr{B}\, p\, p_0\, \frac{\eta_\nu}{<\eta_\nu>} <\eta_\nu>$$

The strength

$$\mathscr{B} = \frac{g_A^2}{6} \left| < {}^{187}Os(1/2^-) \| \sqrt{\frac{4\pi}{3}} \sum_n \tau_n^+ \frac{r_n}{R} \{\sigma_n \otimes Y_1(\Omega_{r_n})\}_2 \| {}^{187}Re(5/2^+) > \right|^2$$

$T_{1/2} = 4.35 \times 10^{10}$ y $\Rightarrow$ $\mathscr{B} = 3.57 \times 10^{-4}$ $\Gamma^\nu({}^{187}Re) = 2.75\ 10^{-32}\ y^{-1}$

Ratio of capture and decay rates

$$\frac{\Gamma^\nu({}^{187}Re)}{\Gamma^\beta({}^{187}Re)} = 1.7\ 10^{-21}$$

MARE: 760 g of $AgReO_4$ bolometers $\Rightarrow$

200 larger as for ^3H

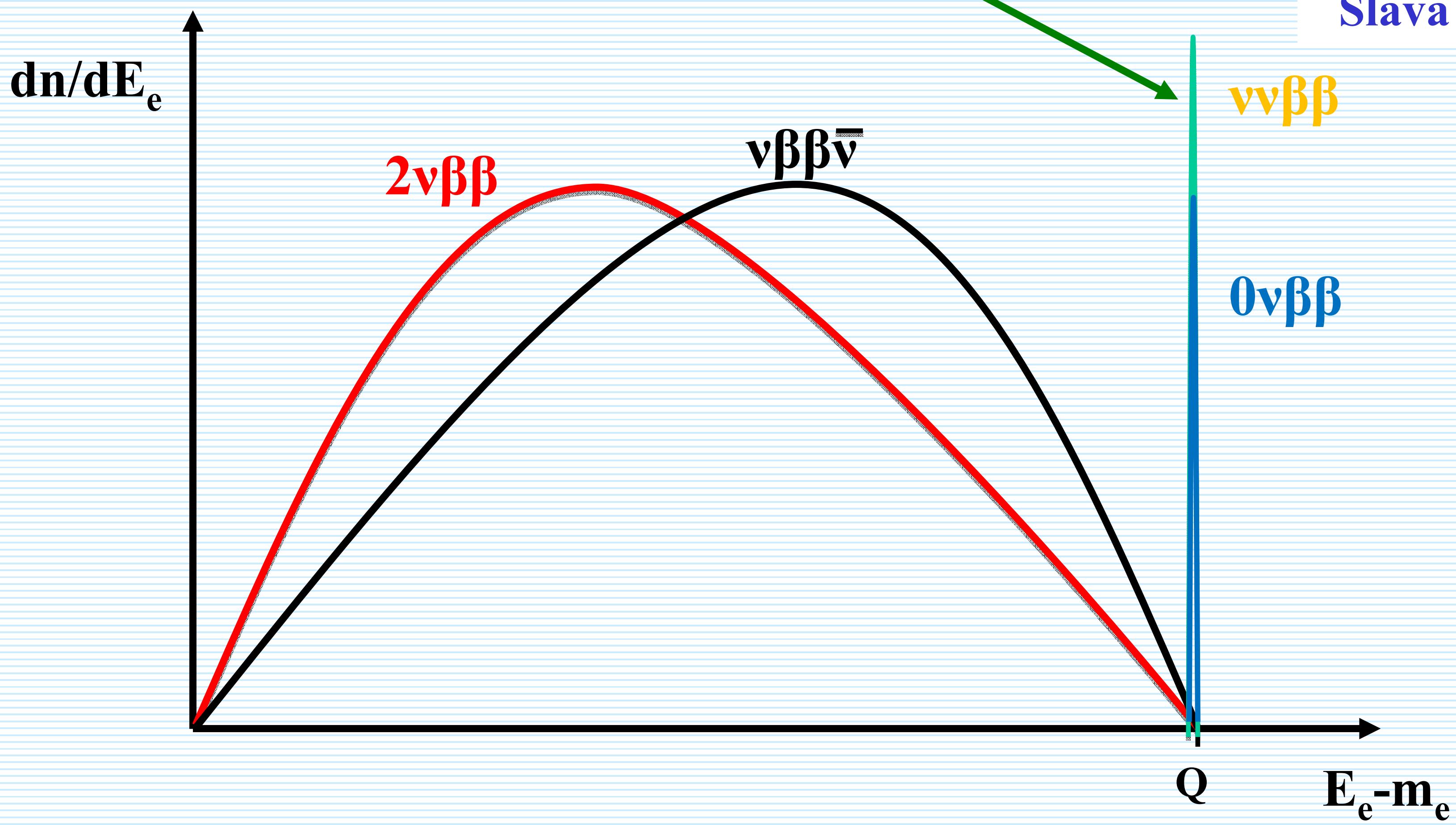
$$N^\nu_{capt}(MARE) \simeq 7.6\ 10^{-8} \frac{\eta_\nu}{<\eta_\nu>}\ y^{-1}$$

50 smaller as for ^3H

Double capture of relic neutrinos by DBD experiment

$\nu_e + \nu_e + (A,Z) \rightarrow (A,Z+2) + e^- + e^-$

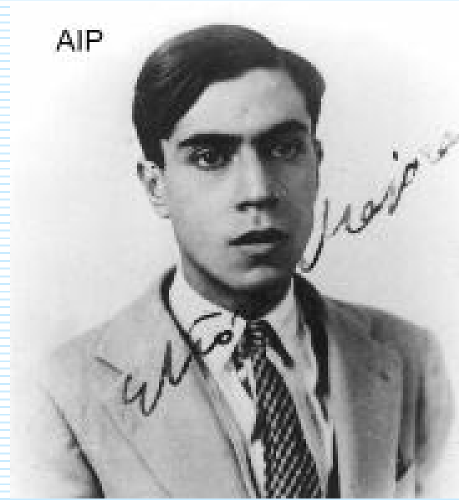
A question of
Slava Eqorov …



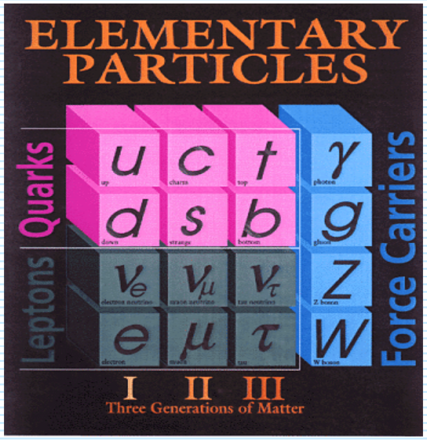
What is the nature of neutrinos?

$\nu \quad \Rightarrow$

GUT's



Only the 0νββ-decay can answer this fundamental question



Standard Model Lepton Universality

Particle	*Symbol*	*Anti – p.*	*mass* [MeV]	L_e	L_μ	L_τ	*life – time* [s]
electron	e^-	e^+	0.511	1	0	0	*stable*
el.neutrino	ν_e	$\overline{\nu}_e$	$< 2.2\ 10^{-6}$	1	0	0	*stable*
muon	μ^-	μ^+	105.6	0	1	0	$2.2\ 10^{-6}$
muon neutr.	ν_μ	$\overline{\nu}_\mu$	< 0.19	0	1	0	*stable*
tau	τ^-	τ^+	1777.	0	0	1	$2.9\ 10^{-13}$
tau neutrino	ν_τ	$\overline{\nu}_\tau$	< 18.2	0	0	1	*stable*

Lepton Family Number Violation

NEW PHYSICS

massive neutrinos, SUSY…

Total Lepton Number Violation

$\nu_{e,\mu\tau} \leftrightarrow \nu_{e,\mu\tau}$, $\overline{\nu}_{e,\mu\tau} \leftrightarrow \overline{\nu}_{e,\mu\tau}$	*observed*
$\mu^+ \rightarrow e^+ + \gamma$	$R \leq 1.2 \times 10^{-11}$
$\mu^+ \rightarrow e^+ + e^- + e^+$	$R \leq 1.0 \times 10^{-12}$
$K^+ \rightarrow \pi^+ + e^- + \mu^+$	$R \leq 4.7 \times 10^{-12}$
$\tau^- \rightarrow e^- + \mu^+ + \mu^-$	$R \leq 1.8 \times 10^{-6}$
$Z^0 \rightarrow e^\pm + \mu^\mp$	$R \leq 1.7 \times 10^{-6}$
$\mu_b^- + (A,Z) \rightarrow (A,Z) + e^-$	$R \leq 1.2 \times 10^{-11}$

$\nu_{e,\mu\tau} \leftrightarrow \overline{\nu}_{e,\mu\tau}$	*not observed*
$K^+ \rightarrow \pi^- + e^+ + \mu^+$	$R \leq 5 \times 10^{-10}$
$\tau^- \rightarrow \pi^- + \pi^+ + e^+$	$R \leq 1.9 \times 10^{-6}$
$W^- + W^- \rightarrow e^- + e^-$	
$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$	$T^{0\nu} \geq 1.9 \times 10^{-25}$
$\mu_b^- + (A,Z) \rightarrow (A,Z-2) + e^+$	$R \leq 3.6 \times 10^{-11}$
$e^- + e^- \rightarrow \pi^- + \pi^-$	?

ν oscillations proposed by Bruno Pontecorvo in Dubna in 1957

The answer to the question whether neutrinos are their own antiparticles is of central importance, not only to our understanding of neutrinos, but also to our understanding of the origin of mass.

$$\frac{1}{T_{1/2}} = G^{0\nu}(E_0, Z)|M'^{0\nu}|^2|\langle m_{\beta\beta}\rangle|^2 \ ,$$

$$m_{\beta\beta} = \sum_{i=1}^{3} U_{ei}^2 \ m_i$$

Absolute ν mass scale

Normal or inverted Hierarchy of ν masses

CP-violating phases

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

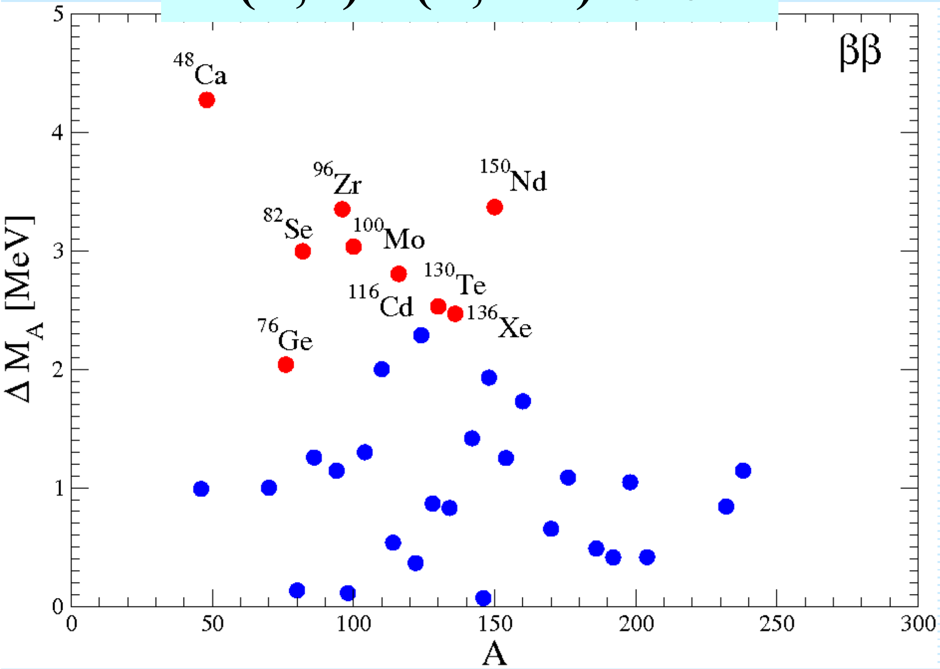
$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

An accurate knowledge of the nuclear matrix elements, which is not available at present, is however a pre-requisite for exploring neutrino properties.

The double beta decay process can be observed due to nuclear pairing interaction that favors energetically the even-even nuclei over the odd-odd nuclei

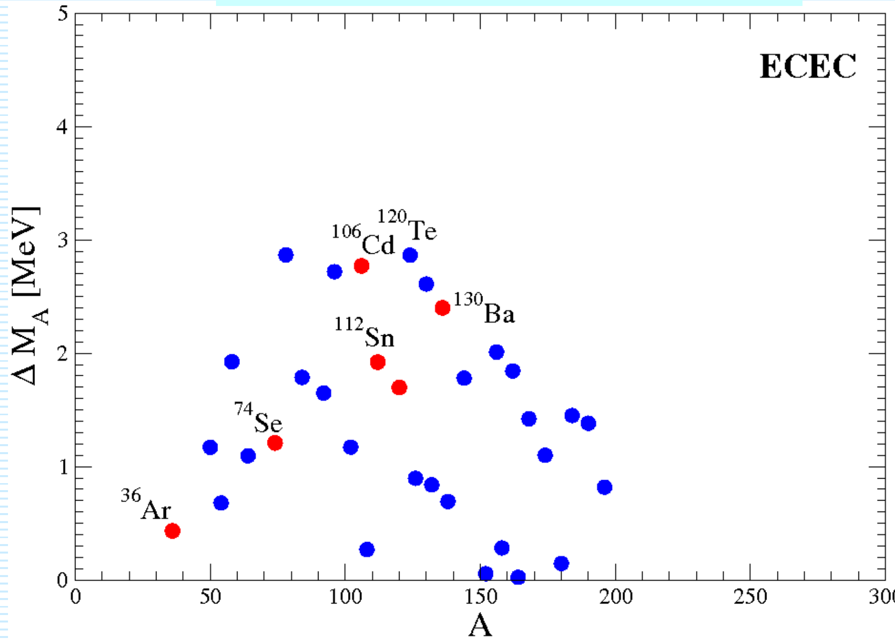
Emission of two electrons

$(A,Z) \rightarrow (A,Z+2)+e+e$



Double electron capture

$e_b+e_b+(A,Z) \rightarrow (A,Z)^*$



Preferable nuclear systems with large ΔM_A (E^5)

Nuclear systems with small ΔM_A might be also important (resonant enhancement)

Signal from γ- and X-rays

The $0\nu\beta\beta$-decay NMEs

In double beta decay two neutrons bound in the ground state of an initial even-even nucleus are simultaneously transformed into two protons that are bound in the ground state or excited (0^+, 2^+) states of the final nucleus

It is necessary to evaluate, with a sufficient accuracy, wave functions of both nuclei, and evaluate the matrix element of the $0\nu\beta\beta$-decay operator connecting them

This can not be done exactly, some approximation and/or truncation is always needed. Moreover, there is no other analogues observable that can be used to judge the quality of the result.

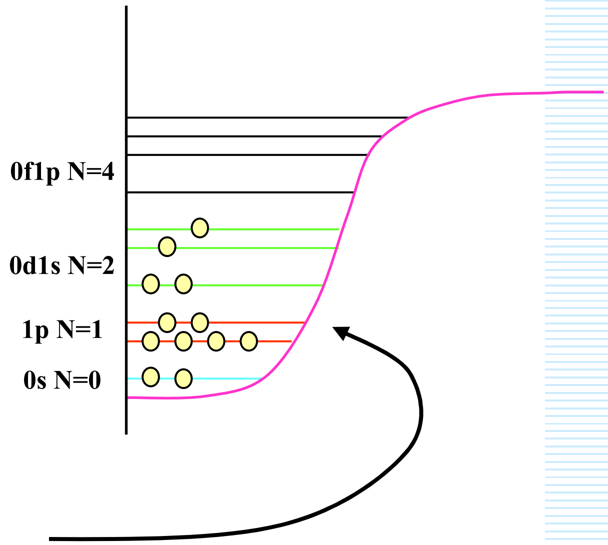
Many-body Hamiltonian

- **Start with the many-body Hamiltonian**

$$H = \sum_i \frac{\vec{p}_i^{\,2}}{2m} + \sum_{i<j} V_{NN}\left(\vec{r}_i - \vec{r}_j\right)$$

- **Introduce a mean-field *U* to yield basis**

$$H = \sum_i \left(\frac{\vec{p}_i^{\,2}}{2m} + U(r_i) \right) + \underbrace{\sum_{i<j} V_{NN}\left(\vec{r}_i - \vec{r}_j\right) - \sum_i U(r_i)}_{\text{Residual interaction}}$$



The success of any nuclear structure calculation depends on the choice of the mean-field basis and the residual interaction!

- **The mean field determines the shell structure**
- **In effect, nuclear-structure calculations rely on perturbation theory**

Two complementary procedures are commonly used:

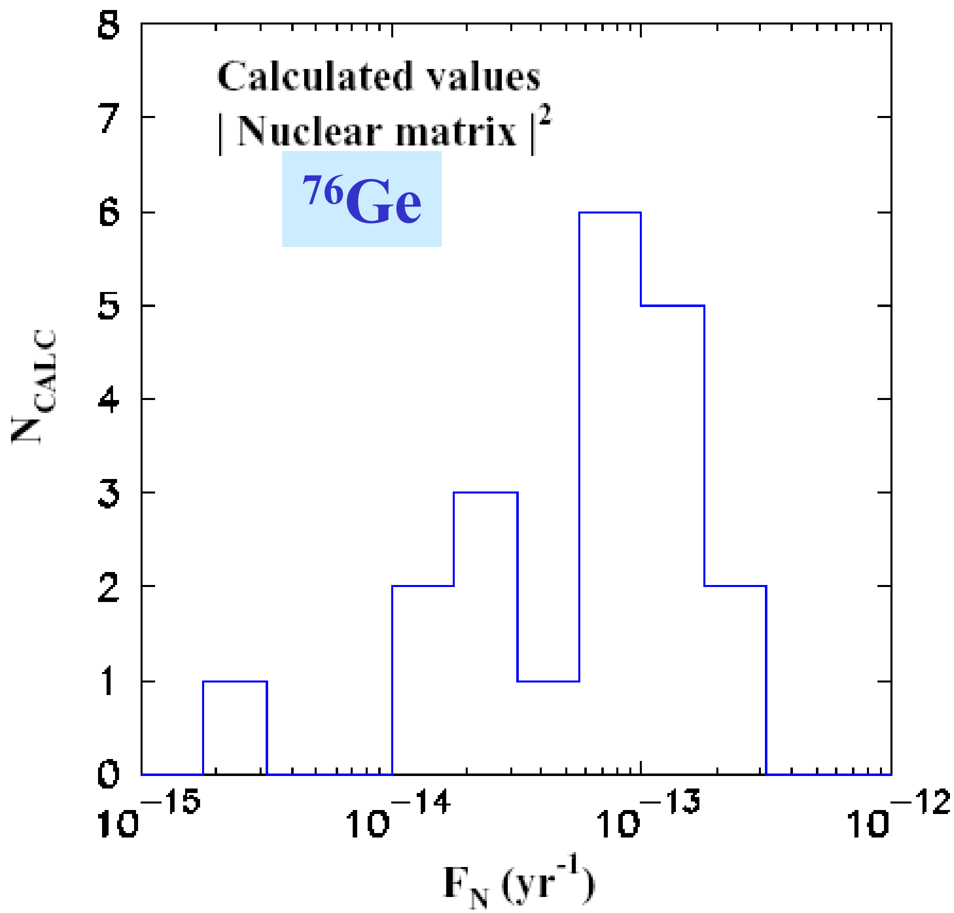
- **Nuclear shell model (NSM)**
- **Quasiparticle Random Phase Approximation (QRPA)**

In NSM a limited valence space is used but all configurations of valence nucleons are included. Describes well properties of low-lying nuclear states. Technically difficult, thus only few $0\nu\beta\beta$-decay calculations

In QRPA a large valence space is used, but only a class of configurations is included. Describe collective states, but not details of dominantly few particle states. Relative simple, thus more 0nbb-decay calculations

Differences: - mean field
- residual interaction
- size of the model space
- many-body approximation

A simple view on the problem of the NMEs (2004)



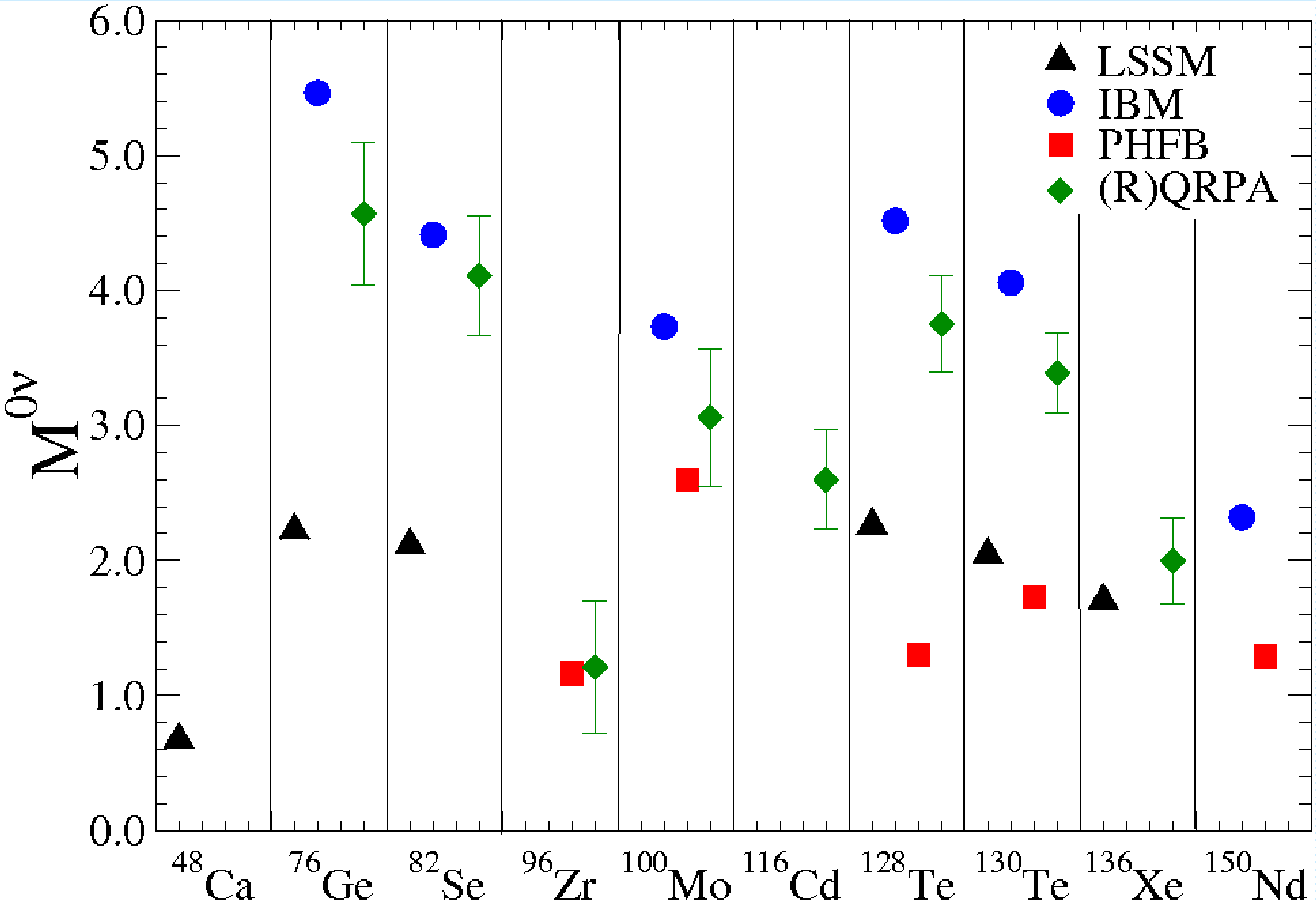
This suggest an uncertainty of NME as much as **factor 5 !!**

Uncertainties in $0\nu\beta\beta$–decay NME?

Is it really so bad?!

Bahcall, Murayama, Pena-Garay, Phys. Rev. D 70, 033012 (2004)

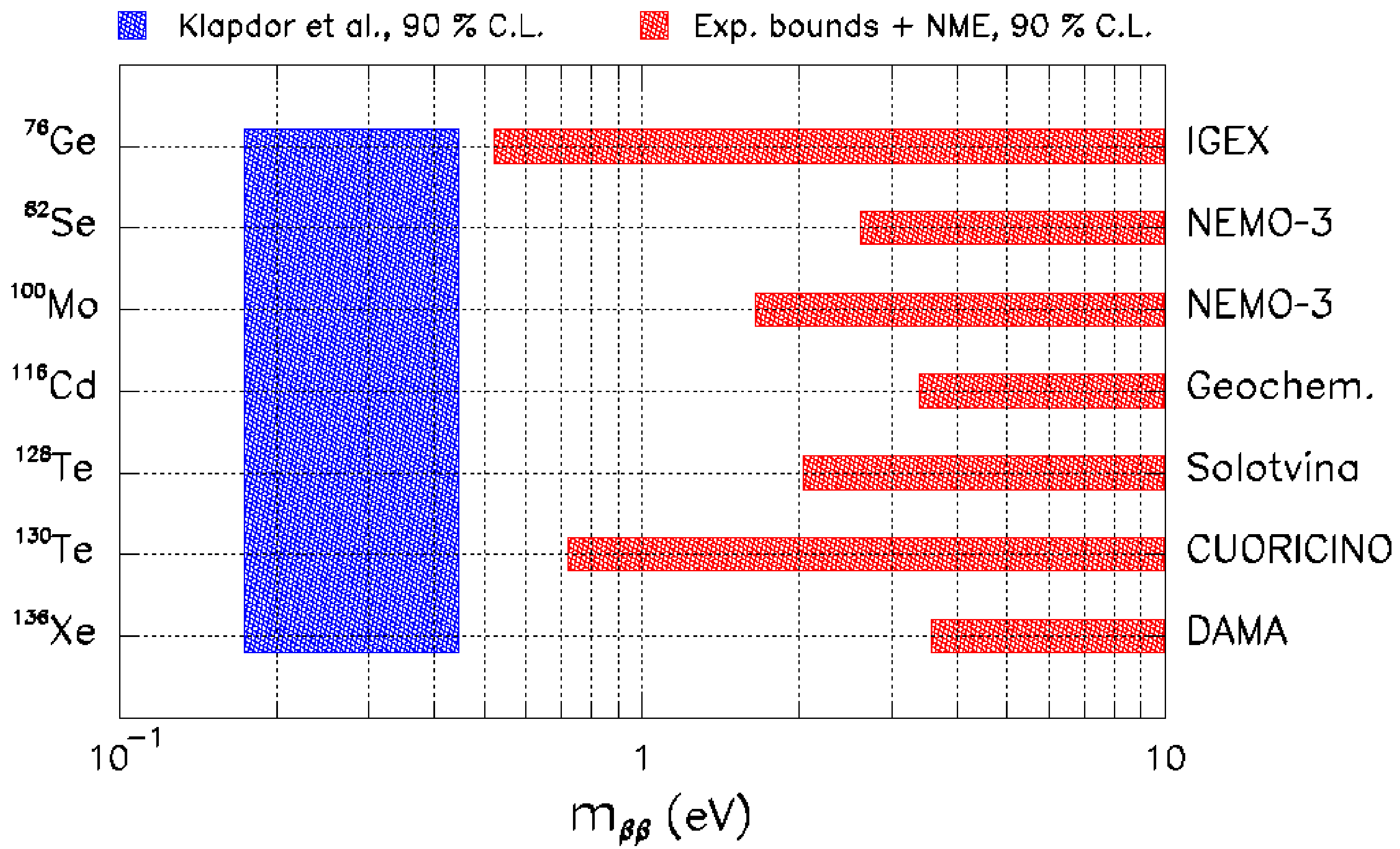
The 0νββ-decay NMEs (Status:2009)



Nobody is perfect:

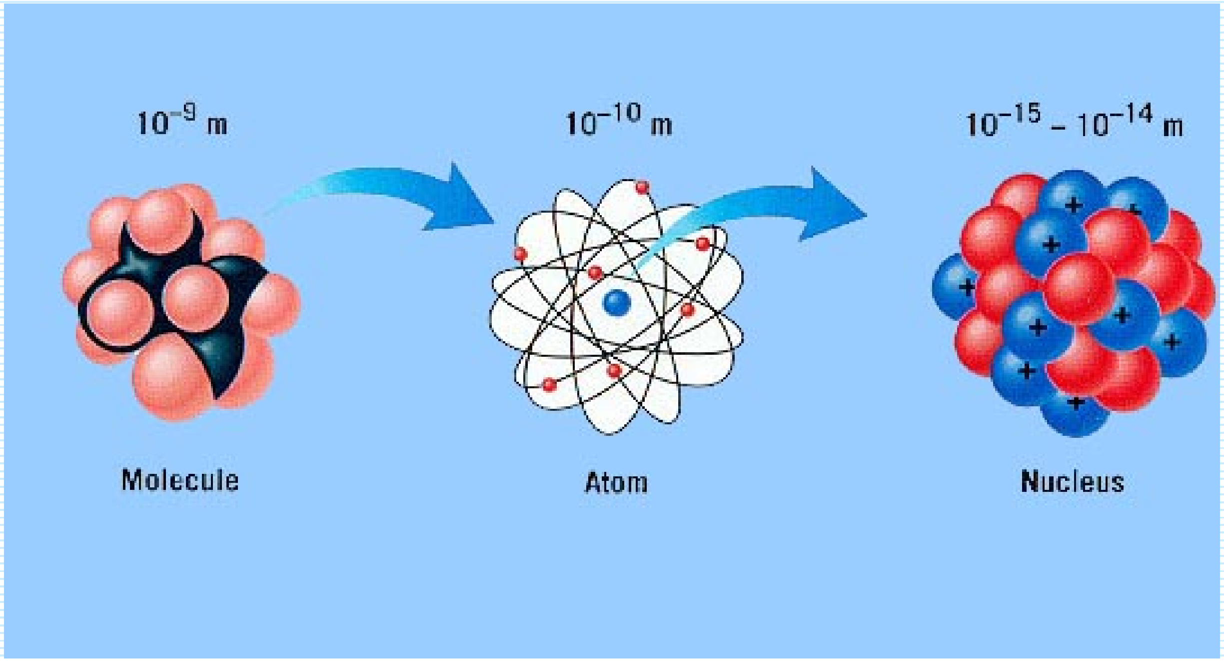
LSSM (small m.s., negative parity states)
PHFB (GT force neglected)
IBM (Hamiltonian truncated)
(R)QRPA (g.s. correlations not accurate enough)

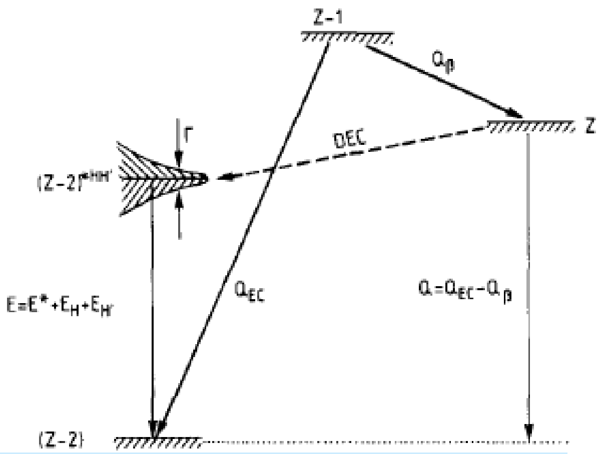
A claim of evidence and other experiments (current status, QRPA NMEs)



Faessler, Figli, Lisi, Rodin, Rotunno, F.Š., PRD 79, 053001 (2009)

Neutrinoless Double Electron Capture





Neutrinoless double eleectron capture
(resonace transitions)
(A,Z)→(A,Z-2)$^{*HH'}$

J. Bernabeu, A. DeRujula, C. Jarlskog,
Nucl. Phys. B 223, 15 (1983)

DEC transitions, abundance, daughter nuclear excitation, atomic vacancies and figure of merit of some isotopes [10]

Transition $Z \to Z-2$	Z-natural abundance in %	Nuclear excitation E^* (in MeV), J^P	Atomic vacancies H, H′	Figure of merit $Q-E$ (in keV)
$^{74}_{34}$Se → $^{74}_{32}$Ge	0.87	1.204 (2^+)	2S(P), 2S(P)	2 ± 3
$^{78}_{36}$Kr → $^{78}_{34}$Se	0.36	2.839 (2^+) 2.864 (?)	1S, 1S	$^{19}_{-6} \pm 10$
$^{102}_{46}$Pd → $^{102}_{44}$Ru	1	1.103 (2^+) 1.107 (4^+)	1S, 1S	$^{29}_{25} \pm 9$
$^{106}_{48}$Cd → $^{106}_{46}$Pd	1.25	2.741 (?)	1S, 1S	-8 ± 10
$^{112}_{50}$Sn → $^{112}_{48}$Cd	1.01	1.871 (0^+)	1S, 1S	-3 ± 10
$^{130}_{56}$Ba → $^{130}_{54}$Xe	0.11	2.502 (?) 2.544 (?)	1S, 1S 1S, 2S(P)	$^{8}_{-6} \pm 13$
$^{152}_{64}$Gd → $^{152}_{62}$Sm	0.20	0 (0^+)	1S, 2S	4 ± 4
$^{162}_{68}$Er → $^{162}_{66}$Dy	0.14	1.783 (2^+)	1S, 2S	1 ± 6
$^{164}_{68}$Er → $^{164}_{66}$Dy	1.56	0 (0^+)	2S, 2S	9 ± 5
$^{168}_{70}$Yb → $^{168}_{68}$Er	0.14	1.355 (1^-) 1.393 (?)	1S, 2S 2S, 2S	$^{-1}_{8} \pm 4$
$^{180}_{74}$W → $^{180}_{72}$Hf	0.13	0 (0^+) 0.093 (2^+)	1S, 1S 1S, 3S	$^{26}_{-4} \pm 17$
$^{196}_{80}$Hg → $^{196}_{78}$Pt	0.15	0.689 (2^+)	1S, 2S	26 ± 9

Atom mixing amplitude
ΔM

$$E \simeq E^* + E_{\mathrm{H}} + E_{\mathrm{H'}},$$

$$\Gamma \simeq \Gamma^* + \Gamma_{\mathrm{H}} + \Gamma_{\mathrm{H'}}.$$

Decay rate

$$\frac{1}{\tau} \simeq \frac{(\Delta M)^2}{(Q-E)^2 + \frac{1}{4}\Gamma^2}\,\Gamma,$$

2νECEC-background depends strongly on Q-value

Different types of Oscillations (Effective Hamiltonian)

Oscillations of ν_l-$\nu_{l'}$
(lepton flavor)

$$H_{eff}^{K_0\overline{K_0}} = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \Gamma_{12} \\ M_{12}^* - \Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of K_0-anti{K_0}
(strangeness)

$$H_{eff}^{n\overline{n}} = \begin{pmatrix} M & V^{BNV} \\ V^{BNV} & M - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of n-anti{n}
(baryon number)

$$H_{eff}^{atom} = \begin{pmatrix} M_i & V^{LNV} \\ V^{LNV} & M_f - \frac{i}{2}\Gamma \end{pmatrix}$$

Oscillation of Atoms (OoA)
(total lepton number)

F.Š., M. Krivoruchenko, Phys.Part.Nucl.Lett. 6 (2009) 485.

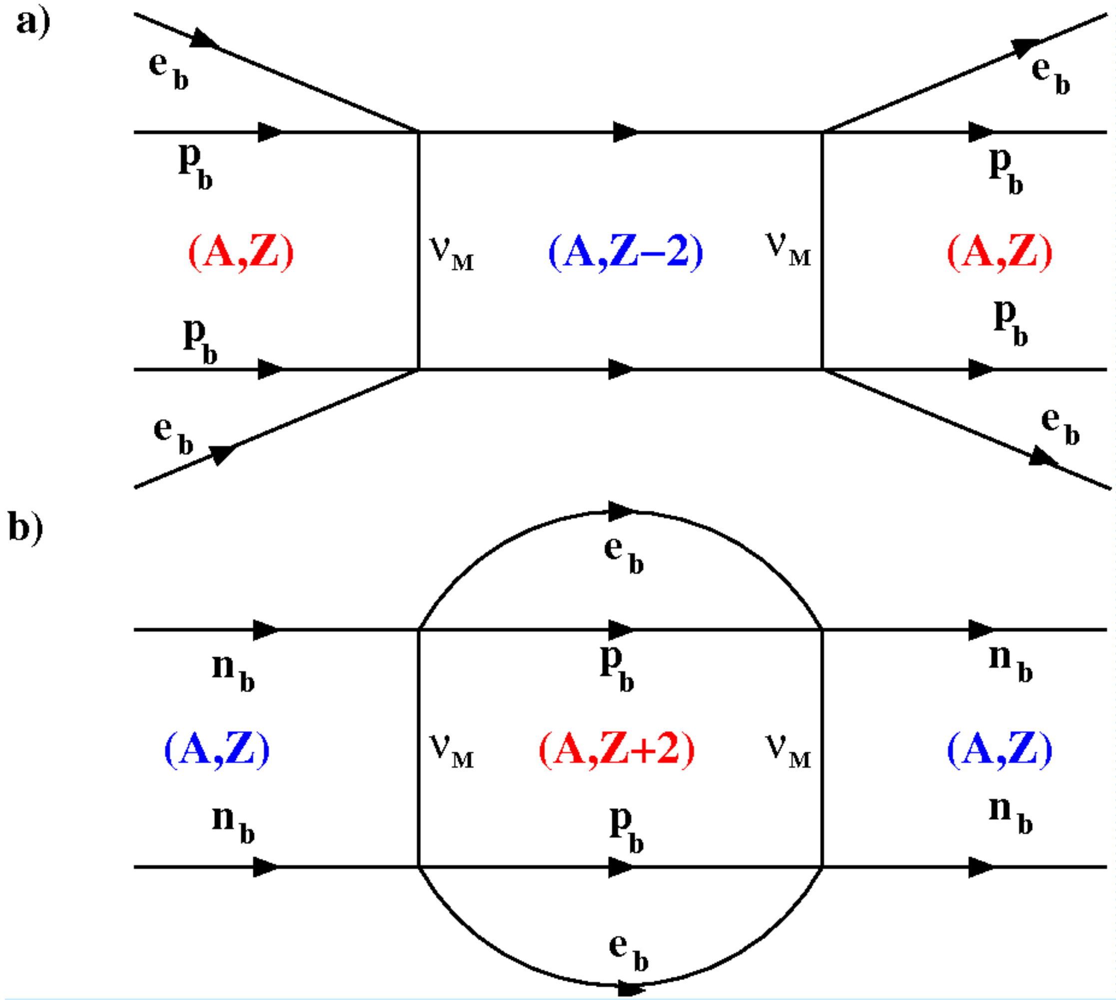
Full width of unstable atom/nucleus

Eigenvalues

$$\lambda_+ = M_i + \Delta M - \frac{i}{2}\Gamma_1,$$
$$\lambda_- = M_f - \frac{i}{2}\Gamma - \Delta M + \frac{i}{2}\Gamma_1$$

$$\Delta M = \frac{V^2(M_i - M_f)}{(M_i - M_f)^2 + \frac{1}{4}\Gamma^2},$$
$$\Gamma_1 = \frac{V^2\Gamma}{(M_i - M_f)^2 + \frac{1}{4}\Gamma^2}.$$

F.Š., M. Krivoruchenko, Phys.Part.Nucl.Lett. 6 (2009) 485.



Mixing of neutral atoms and total lepton number oscillation

$$n + n \leftrightarrow p + p + e_b^- + e_b^-$$

$$(\mathcal{A},\mathcal{Z}) \Longleftrightarrow (\mathcal{A},\mathcal{Z}+2)^{**}$$

$$(\mathcal{A},\mathcal{Z}) \Longleftrightarrow (\mathcal{A},\mathcal{Z}-2)^{**}$$

LNV Potential

$$V^{LNV} \simeq m_{\beta\beta}\, G_F^2 \;<\frac{1}{4\pi r_a}>\; \Psi_1(0)\Psi_2(0)$$

$$V^{LNV} \sim 10^{-24}\ \mathrm{eV}$$

$$m_\nu = 0.5\ eV, \quad Z = 30, \quad n_i = 1, \quad l_i = 0$$

Oscillations of stable atoms (Γ=0)

$$|<f|e^{-iH_{eff}t}|i>|^2 = \frac{4V^2}{(M_i - M_f)^2} \sin^2[t\ (M_i - M_f)/2]$$

$[t\ (M_i - M_f)] \leq 1$

$$|<f|e^{-iH_{eff}t}|i>|^2 = V^2 t^2$$

$[t\ (M_i - M_f)] \geq 1$

$$|<f|e^{-iH_{eff}t}|i>|^2 \approx \frac{V^2}{(M_i - M_f)^2}$$

$$^{164}_{68}Er \rightarrow\ ^{164}_{66}Dy$$
$$(M_i - M_f) = 24.1\ keV$$

$$|<f|e^{-iH_{eff}t}|i>|^2 \leq 3\ 10^{-55}$$

Double electron capture (Γ≠0)
(resonant enhancement of atom)

$$\begin{aligned}\Gamma &= 4 \times 10^{-7}\ Z^4\ \mathrm{eV} \\ &= 0.3\ eV \qquad (Z = 30)\end{aligned}$$

$$\begin{aligned}R_max &= \frac{1\ \mathrm{ton}}{M_i} \times \frac{4V^2}{\Gamma} \\ &\sim 10^4\ yr^{-1}\end{aligned}$$

Mass difference >> Γ

$$\Gamma_1 = \frac{4V^2}{4(M_i - M_f)^2 + \Gamma^2}\ \Gamma$$

$$R \sim R_{max} \frac{\Gamma^2}{(M_i - M_f)^2} \sim 10^{-3}\ yr^{-1}$$

Mass difference ~ keV

Double electron capture

$e_{1s1/2} + e_{1s1/2} + {}^{112}Sn \rightarrow {}^{112}Cd(0^+_3)$

Reletivistic electron w.f. (j=1/2, l=0, l'=1)

$$\Psi^{(\alpha)}_{jm}(\vec{x}) = \begin{pmatrix} f_\alpha(r)\ \Omega_{jlm} \\ (-1)^{\frac{1+l+l'}{2}} g_\alpha(r)\ \Omega_{jl'm} \end{pmatrix} \quad l = j \pm 1/2,\ l' = 2j - l$$

Potential

0.022

$$V^{1s_{1/2}1s_{1/2}}(0^+_3) = \frac{1}{4\pi}\ m_e\ \left(G^2_\beta m^4_e\right)\ \frac{m_{\beta\beta}}{m_e}\ \frac{1}{R\ m_e}\ \frac{\left(\bar{f}_{1s_{1/2}}\right)^2}{4\pi\ m^3_e} g^2_A\ M^{0\nu}(0^+_3).$$

Width

$$\Gamma^{ECEC} = \frac{\left|V^{1s_{1/2}1s_{1/2}}(0^+_3)\right|^2}{(M_i - M_f)^2 + \frac{\Gamma^2_X}{4}}\ \Gamma_X$$

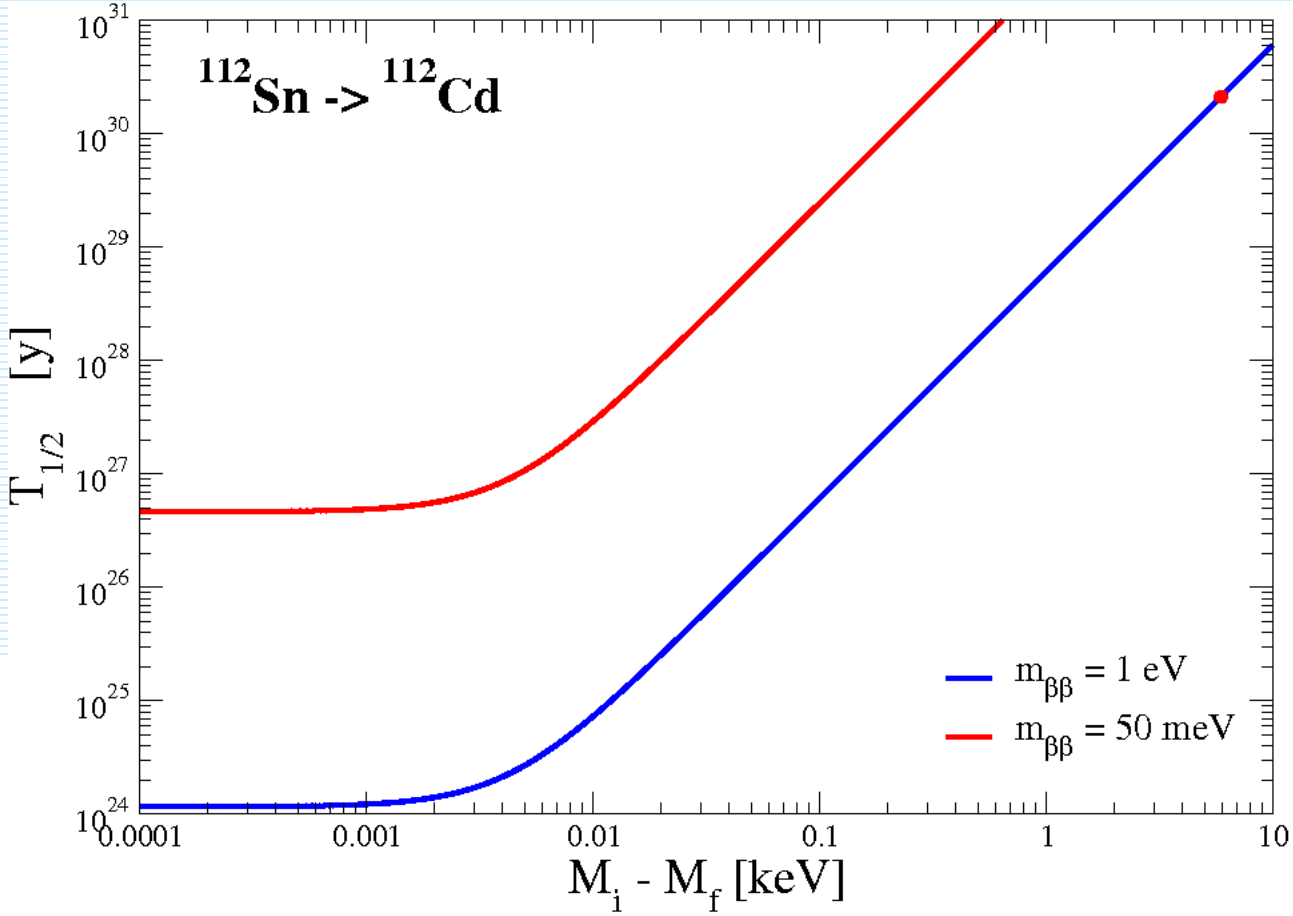
Matrix element

Exc. state	E_{ex} (MeV)	$M^{0\nu}$
$0^+_{g.s.}$	0	2.69
0^+_1 (1 ph.)	1.224	3.02
0^+_2 (2 ph.)	1.433	0.90
0^+_3 (1 ph.)	1.224	2.78

Double electron capture of ^{112}Sn (perspectives of search)

F. Šimkovic, M. Krivoruchenko, A. Faessler, to be submitted

M_i-M_f	$T_{1/2}^{ECEC}$ ($m_{\beta\beta}$= 50 meV)
1 keV	2.44 10^{31} years
100 eV	2.45 10^{29} years
10 eV	2.91 10^{27} years
0 eV	4.67 10^{26} years



No 2νECEC background!

$T^{2\nu ECEC}$ = 1.7 10^{22} y ($0^+_{g.s.}$)

7.4 10^{24} y (0^+_1)

5.4 10^{34} y (0^+_3)

Domin, Kovalenko, F. Š., Semenov, NPA 753, 337 (2005)

$T_{1/2}^{0\nu}$ (^{76}Ge)= (2.95 – 5.74) 10^{26} years for $m_{\beta\beta}$= 50 meV

$J^\pi=0^+$

Calculated double electron capture half-lives ($m_{\beta\beta}$ = 1 eV)

Transition	$M^*_{A,Z-2}-M_{A,Z-2}$	$M^{**}_{A,Z-2}-M_{A,Z}$	Holes	$T^{\min}_{1/2}$	$T_{1/2}$
${}^{112}_{50}\mathrm{Sn}\rightarrow{}^{112}_{48}\mathrm{Cd}^*$	1871 ± 0.2	$-5.9\pm4.2\pm2.7$	$1s_{1/2}\ 1s_{1/2}$	2×10^{24}	8×10^{30}
${}^{152}_{64}\mathrm{Gd}\rightarrow{}^{152}_{62}\mathrm{Sm}$	0	$-0.3\pm2.5\pm2.5$	$1s_{1/2}\ 2s_{1/2}$	5×10^{24}	9×10^{29}
	0	$5.9\pm2.5\pm2.5$	$1s_{1/2}\ 3s_{1/2}$	4×10^{25}	8×10^{29}
	0	$7.4\pm2.5\pm2.5$	$1s_{1/2}\ 4s_{1/2}$	8×10^{26}	10^{33}
${}^{148}_{64}\mathrm{Gd}\rightarrow{}^{148}_{62}\mathrm{Sm}^*$	3045 ± 2	$5.7\pm2.5\pm2.5$	$2s_{1/2}\ 2s_{1/2}$	8×10^{25}	3×10^{32}
	3045 ± 2	$11.8\pm2.5\pm2.5$	$2s_{1/2}\ 3s_{1/2}$	3×10^{26}	8×10^{33}
	3045 ± 2	$13.3\pm2.5\pm2.5$	$2s_{1/2}\ 4s_{1/2}$	4×10^{27}	2×10^{35}
	3045 ± 2	$6.6\pm2.5\pm2.5$	$2p_{1/2}\ 2p_{1/2}$	2×10^{29}	2×10^{36}
${}^{156}_{66}\mathrm{Dy}\rightarrow{}^{156}_{64}\mathrm{Gd}^*$	1988.5 ± 0.2	$7.0\pm6.6\pm2.5$	$2s_{1/2}\ 2s_{1/2}$	2×10^{27}	8×10^{31}
	1988.5 ± 0.2	$7.9\pm6.6\pm2.5$	$2p_{1/2}\ 2p_{1/2}$	8×10^{29}	4×10^{35}

Atomic effects taken into account: repulsion of two holes

Transition	J^P	$M^*_{A,Z-2}-M_{A,Z-2}$	$M^{**}_{A,Z-2}-M_{A,Z}$	Holes	$T^{\min}_{1/2}$	$T_{1/2}$
${}^{162}_{68}\mathrm{Er}\rightarrow{}^{162}_{66}\mathrm{Dy}^*$	1^+	1745.716 ± 0.007	$-10.1\pm3.5\pm2.5$	$1s_{1/2}\ 1s_{1/2}$	8×10^{23}	2×10^{29}
${}^{156}_{66}\mathrm{Dy}\rightarrow{}^{156}_{64}\mathrm{Gd}^*$	1^+	1965.950 ± 0.004	$-12.5\pm6.6\pm2.5$	$1s_{1/2}\ 2s_{1/2}$	10^{25}	3×10^{30}
	1^+	1965.950 ± 0.004	$-5.8\pm6.6\pm2.5$	$1s_{1/2}\ 3s_{1/2}$	2×10^{26}	2×10^{31}
	1^-	1946.375 ± 0.006	$8.4\pm6.6\pm2.5$	$1s_{1/2}\ 2s_{1/2}$	8×10^{26}	4×10^{31}
${}^{74}_{34}\mathrm{Se}\rightarrow{}^{74}_{32}\mathrm{Ge}^*$	2^+	1204.204 ± 0.007	$3.0\pm1.7\pm1.6$	$2p_{1/2}\ 2p_{3/2}$	10^{36}	10^{45}

Lepton number and parity oscillations

$0^+\rightarrow2^+$ strongly suppressed, $p_{3/2}$-electron needed (squared R/a_B-factor)

Q-value measurements
Klaus Blaum "LAUNCH09 (Nov. 09)"

Accuracy below 300 eV is not a problem

Decay	Q-value	Precision
	$\beta\beta$	
$^{76}Ge - ^{76}Se$	2039.006(50) G. Douysset et al., PRL 86, 4259 (2001)	6E-10
$^{130}Te - ^{130}Xe$	2527.518(13) M. Redshaw et al., PRL 102, 212502 (2009)	1E-10
$^{136}Xe - ^{136}Ba$	2457.83(37) M. Redshaw et al., PRL 98, 053003 (2007)	3E-09
	ECEC	
$^{112}Sm - ^{112}Cd$	1919.82(16) S. Rahaman et al., PRL 103, 042501 (2009)	1E-09
$^{120}Te - ^{120}Sm$	1714.81(1.25) N. Scielzo et al., PRC 80, 025501 (2009)	1E-08

Is it possible to manipulate atomic mass difference?

Magnetic field of 10 T would be not enough …

$2\nu\beta\beta$-decay and statistical properties of ν

Mixed statistics for neutrinos

Definnition of mixed state

$$\begin{aligned} |\nu> &= \hat{a}^\dagger|0> \\ &\equiv \cos\delta\ \hat{f}^\dagger|0> \ + \ \sin\delta\ \hat{b}^\dagger|0> \\ &= \cos\delta\ |f> + \ \sin\delta\ |b> \end{aligned}$$

with commutation Relations

$$\begin{aligned} \hat{f}\hat{b} &= e^{i\phi}\hat{b}\hat{f} \qquad \hat{f}^\dagger\hat{b}^\dagger = e^{i\phi}\hat{b}^\dagger\hat{f}^\dagger \\ \hat{f}\hat{b}^\dagger &= e^{-i\phi}\hat{b}^\dagger\hat{f} \qquad \hat{f}^\dagger\hat{b} = e^{-i\phi}\hat{b}\hat{f}^\dagger \end{aligned}$$

Amplitude for 2νββ

$$\begin{aligned} A^{2\nu} &= [\cos\delta^4 + \cos\delta^2\sin\delta^2(1-\cos\phi)]A^f \ + \ [\cos\delta^4 + \cos\delta^2\sin\delta^2(1+\cos\phi)]A^b \\ &= \cos\chi^2 A^f \ + \ \sin\chi^2 A^b \end{aligned}$$

Decay rate

$$\begin{aligned} W^{2\nu} &= \cos\chi^4\ W^f \ + \ \sin\chi^4\ W^b \\ &= (1-b^2)\ W^f \ + \ b^2\ W^b \end{aligned}$$

Partly bosonic neutrino requires knowing NME or log ft values for HSD or SSD

(calculations coming up soon)

Looking for a signature of bosonic ν

2νββ–decay half-lives ($0^+ \to 0^+_{g.s.}$, $0^+ \to 0^+_1$, $0^+ \to 2^+_1$)

- HSD – NME needed
- SSD – log ft_{EC}, log ft_{β} needed

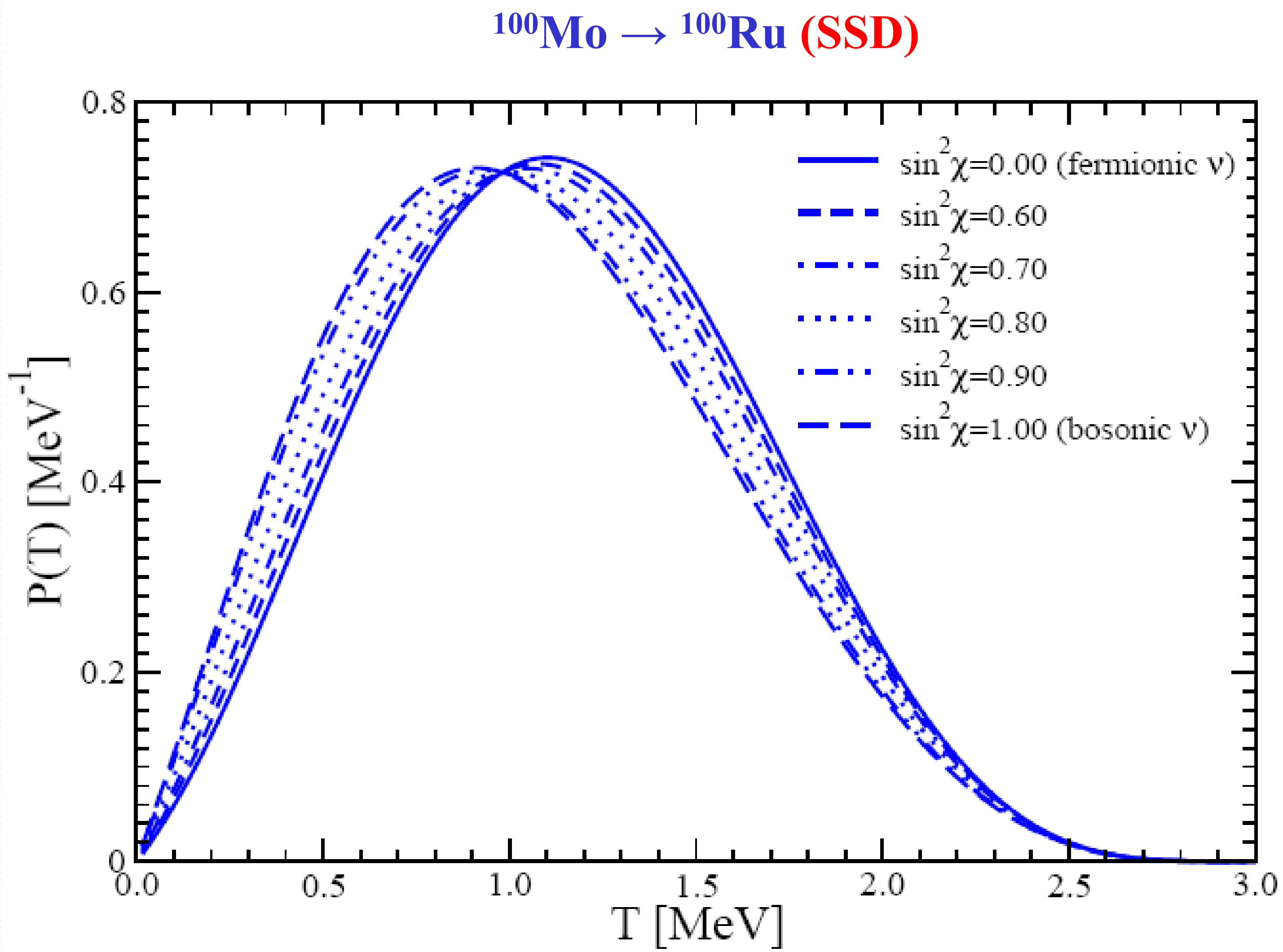
$$
\begin{aligned}
\frac{T_{1/2}^{2\nu-SSD}(2^+_f)}{T_{1/2}^{2\nu-SSD}(0^+_f)} &= 2.41 \times 10^4 \quad \text{fermionic } \nu \qquad & T_{1/2}^{2\nu}(2^+) &= 1.73 \times 10^{23} years \\
&= 403 \quad \text{bosonic } \nu & &= 2.74 \times 10^{21} years \\
& & T_{1/2}^{2\nu-exp}(2^+) &> 1.6 \times 10^{21} years
\end{aligned}
$$

Normalized differential characteristics

- The single electron energy distribution
- The distribution of the total energy of two electrons
- Angular correlations of two electrons

(free of NME and log ft)

Mixed ν excluded for $\sin^2\chi < 0.6$



Summary

• **Single β-decay of ^{3}H and ^{187}Re**

Relativistic calculation for β-decay of ^{3}H presented by considering also the nuclear recoil. The spectrum of the β-decay of ^{187}Re presented. Practically, the emitted electrons are only in $p_{3/2}$ states.

• **Detection of relic neutrinos with KATRIN and MARE**

Even in the case of clustering of neutrinos the production rate is small $\approx 10^{-3(-4)}$ per year. Possibility to establish upper limit on the density of relic neutrinos.

• **The $0\nu\beta\beta$-decay NMEs**

Significant progress achieved. But, further studies needed. (Effects of deformation, many-body approximations … Uncertainties …)

• **Neutrinoless double electron capture**

Proposed OoA. A phenomenological analysis of this process lead to a resonant enhancement of the DEC that has a Breit-Wigner form.

• **$2\nu\beta\beta$-decay energy distribution allows to conclude whether neutrinos obey Bose-Einstein or Fermi-Dirac statistics**