Origin of the Lorentzian metric, standard supersymmetry, and Higgs fields

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The main new ideas in this talk have not been presented or published elsewhere, but they build upon work presented in earlier papers:

R. E. Allen, Int. J. Mod. Phys. A 12, 2385 (1997); hep-th/9612041.
R. E. Allen, Proceedings of Beyond the Desert 2002; hep-th/0008032.
R. E. Allen, Proceedings of Beyond the Desert 2003; hep-th/0310039.

In some of this earlier work we suggested the possibility of spin 1/2 bosons, which would certainly be a dramatic prediction for the LHC. Here we show that the present theory leads more plausibly to standard supersymmetry with the usual spin 0 sfermions, spin 1/2 gauginos, etc. The new predictions of the present theory for experiment are thus going to be more subtle -- but the present theory is still relatively close to real-world physics, and there are certainly new predictions at sufficiently high energies and on large enough distance scales.

The present theory demonstrates the robustness of the predictions of standard supersymmetry, true Higgs bosons, supersymmetric candidates for dark matter, and GUT phenomena like neutrinoless double beta decay, and further strengthens our belief that they should be seen or confirmed experimentally within the foreseeable future.

Outline of this talk:

- **origin of the Lorentzian form (-,+,+,+) for the metric tensor, starting with the original Euclidean form.**
- **origin of standard supersymmetry and Higgs fields**
- **vanishing of the usual cosmological constant**
- **successes of the present theory, and work remaining**

The present theory starts with a statistical picture, and initially yields a Euclidean path integral Z involving a Euclidean operator $\widehat{A}\,(x)$. First we will show that the path integral can be re-expressed in Lorentzian form. Then we will find that, in the present theory and at low energy (compared to the Planck scale), the relevant operators also automatically turn out to have Lorentzian form.

Consider the Euclidean action for a multicomponent field Ψ:

$$S = \int d^4x\, \Psi^\dagger\,(x)\; \widehat{A}\,(x)\; \Psi\,(x) \quad .$$

The path integral also has the Euclidean form

$$Z_f = \int \mathcal{D}\,\Psi_f\; \mathcal{D}\,\Psi_f^\dagger\, e^{-S_f}$$

in the case of fermions, with Ψ_f composed of anticommuting Grassmann variables, or

$$Z_b = \int \mathcal{D}\,(\mathrm{Re}\,\Psi_b)\;\; \mathcal{D}\,(\mathrm{Im}\,\Psi_b)\; e^{-S_b}$$

in the case of bosons, with Ψ_b composed of ordinary commuting variables.

Since $\widehat{A}$ contains covariant derivatives, let us work in a locally inertial coordinate system and in the temporal gauge $A_0^i = 0$, so that the states which diagonalize the Hermitian operator $\widehat{A}\,(x)$ have well-defined frequencies ω:

$$\Psi\,(x) = \sum_{\omega,r} b_r\,(\omega)\,\Psi_r\,(\overrightarrow{x})\,e^{-i\,\omega\,x^0} \;\;,\;\; \widehat{A}\,(x)\;\Psi_r\,(\overrightarrow{x})\,e^{-i\,\omega\,x^0} = a\,(\omega,r)\;\Psi_r\,(\overrightarrow{x})\,e^{-i\,\omega\,x^0}$$

where $\overrightarrow{x}$ represents all coordinates except the time x^0, and r distinguishes the states for a given ω. The basis functions $\Psi_r\,(\overrightarrow{x})$ are normalized such that this is a unitary transformation, and the Jacobian in the path integral is consequently equal to unity. Then

$$S = \sum_{\omega,r} b_r^\dagger\,(\omega)\;a\,(\omega,r)\;b_r\,(\omega)$$

and we can write

$$Z = \prod_{\omega,r} z\,(\omega,r) = \prod_{\omega>0,r} z\,(\omega,r)\cdot\prod_{\omega<0,r} z\,(\omega,r)\;\;.$$

Start with bosons, for which

$$\begin{aligned}
z_b(\omega, r) &= \int_{-\infty}^{\infty} d(\operatorname{Re} b_r(\omega)) \int_{-\infty}^{\infty} d(\operatorname{Im} b_r(\omega))\, e^{-a(\omega,r)\left[(\operatorname{Re} b_r(\omega))^2 + (\operatorname{Im} b_r(\omega))^2\right]} \\
&= \frac{\pi}{a(\omega, r)}
\end{aligned}$$

(where the b_r and $a(\omega, r)$ are for the subset of states relevant to the bosons). Since $\widehat{A}$ is a Euclidean operator, all of the $a(\omega, r)$ are positive. For $\omega > 0$, let

$$\begin{aligned}
z_b'(\omega, r) &\equiv i \int_{-\infty}^{\infty} d(\operatorname{Re} b_r(\omega)) \int_{-\infty}^{\infty} d(\operatorname{Im} b_r(\omega))\, e^{ia(\omega,r)\left[(\operatorname{Re} b_r(\omega))^2 + (\operatorname{Im} b_r(\omega))^2\right]} \\
&= \frac{\pi}{a(\omega, r)}
\end{aligned}$$

since $\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \exp\left(ia\left(x^2 + y^2\right)\right) = \pi/(ia)$. And for $\omega < 0$ let

$$\begin{aligned}
z_b'(\omega, r) &\equiv -i \int_{-\infty}^{\infty} d(\operatorname{Re} b_r(\omega)) \int_{-\infty}^{\infty} d(\operatorname{Im} b_r(\omega))\, e^{-i\, a(\omega,r)\left[(\operatorname{Re} b_r(\omega))^2 + (\operatorname{Im} b_r(\omega))^2\right]} \\
&= \frac{\pi}{a(\omega, r)}\,.
\end{aligned}$$

We therefore have

$$Z_b = \prod_{\omega>0,r} z_b'(\omega, r) \cdot \prod_{\omega<0,r} z_b'(\omega, r)$$

or

$$Z_b = \prod_{\omega,r} \overline{z}_b(\omega, r)$$

$$\overline{z}_b(\omega, r) \equiv \int_{-\infty}^{\infty} d(\mathrm{Re}\, b_r(\omega)) \int_{-\infty}^{\infty} d(\mathrm{Im}\, b_r(\omega))\, e^{i\, sgn(\omega)\, a(\omega,r)\left[(\mathrm{Re}\, b_r(\omega))^2 + (\mathrm{Im}\, b_r(\omega))^2\right]}$$

since each pair of positive and negative frequencies gives a factor $(-i)\, i = 1$. This is the path integral for an arbitrary time interval, so for any physical process the new expression for Z_b will give the same results as the old expression.

For fermions the procedure is essentially the same, except that the integrals are carried out formally using the rules for Grassmann variables, and we do not separate out positive and negative frequencies.

We can now transform back to the original fields and write the path integral as

$$Z = \int \mathcal{D}\,\Psi\ \mathcal{D}\,\Psi^{\dagger}\ e^{iS}$$

where this form is just schematic for bosons, with $\mathcal{D}\,\Psi_{\,b}\,\mathcal{D}\,\Psi_{\,b}^{\dagger}$ really meaning $\mathcal{D}\,(\mathrm{Re}\,\Psi_{\,b})\ \ \mathcal{D}\,(\mathrm{Im}\,\Psi_{\,b})$, and where

$$S_b = \int d^4x\,\Psi_{+}^{\dagger}\,(x)\ \widehat{A}\,(x)\ \Psi_{+}\,(x) - \int d^4x\,\Psi_{-}^{\dagger}\,(x)\ \widehat{A}\,(x)\ \Psi_{-}\,(x)$$

with

$$\Psi_{+}\,(x) = \sum_{\omega>0,r} b_r\,(\omega)\,\Psi_r\,(\overrightarrow{x}\,)\,e^{-i\,\omega\,x^0} \quad , \quad \Psi_{-}\,(x) = \sum_{\omega<0,r} b_r\,(\omega)\,\Psi_r\,(\overrightarrow{x}\,)\,e^{-i\,\omega\,x^0}\ .$$

Mathematical nuances with Lorentzian path integrals are discussed in Peskin and Schroeder, p. 286. The full Euclidean path integral in the present theory involves nonlinearities which lead to very complicated asymptotic series. However, in a simple treatment of the nonlinear effects the conclusion above is not changed: The Lorentzian and Euclidean path integrals give the same results. Also, the above treatment is a valid aproximation for low field strengths.

Recall that the operator $\widehat{A}$ still has a Euclidean form. However, in the present theory, according to (6.16) of IJMP1997 or (3.45) of Beyond2002, $\widehat{A}$ effectively becomes at low energy (compared to the Planck scale)

$$\widehat{A}' = i\, e_{\alpha}^{\mu}\, \sigma^{\alpha} D_{\mu}$$

where e_{μ}^{α} is the vierbein, with Ψ replaced by ψ, and

$$\Psi = U\, \psi\ .$$

The field ψ is the original field Ψ transformed to the "frame of reference" of the vacuum, which in the present theory contains a GUT condensate that is effectively moving with a "superfluid velocity" corresponding to U. This transformation is the precise generalization of the prescription for giving a Galilean boost to a field in nonrelativistic quantum mechancs, in order to bring it into a moving frame of reference.

For bosons we then have

$$
\begin{aligned}
S_b &= \int d^4x\, \psi_+^\dagger(x)\;\; i\, e_\alpha^\mu\, \sigma^\alpha D_\mu(x)\, \psi_+ - \int d^4x\, \psi_-^\dagger(x)\;\; i\, e_\alpha^\mu\, \sigma^\alpha D_\mu\, \psi_-(x) \\
&= \int d^4x\, e\, \overline{\psi}_+^\dagger(x)\;\; i\, e_\alpha^\mu\, \sigma^\alpha \widetilde{D}_\mu\, \overline{\psi}_+(x) - \int d^4x\, e\, \overline{\psi}_-^\dagger(x)\;\; i\, e_\alpha^\mu\, \sigma^\alpha \widetilde{D}_\mu\, \overline{\psi}_-(x)
\end{aligned}
$$

with

$$
\overline{\psi}_\pm = e^{-1/2}\, \psi_\pm \; .
$$

Here $e = \det e_\mu^\alpha = \left(-\det g_{\mu\nu}\right)^{1/2}$, where $g_{\mu\nu}$ is the metric tensor, and

$$
D_\mu = \partial_\mu + i A_\mu^i t_i \quad , \quad \widetilde{D}_\mu = D_\mu + e^{-1/2} \partial_\mu e^{1/2} \; .
$$

(In Beyond2002 this is derived in a Lorentzian picture, and in IJMP1997 it is derived in a Euclidean picture with e_α^0 imaginary, but it is easy to see that the derivation still works in the present context, with a Euclidean operator and all the e_α^μ real.)

The above form also holds for fermions, except that it is not necessary to treat positive and negative frequencies separately:

$$S_f = \int d^4x \, \psi_f^\dagger (x) \;\; i \, e_\alpha^\mu \, \sigma^\alpha D_\mu (x) \, \psi_f = \int d^4x \, e \, \overline{\psi}_f^\dagger (x) \;\; i \, e_\alpha^\mu \, \sigma^\alpha \widetilde{D}_\mu \, \overline{\psi}_f (x) \;\; .$$

This is the conventional form for fermions, described by 2-component Weyl spinors, but it is highly unconventional for bosons, because a boson described by $\overline{\psi}$ would have spin 1/2.

For bosons, however, we can transform from the set $\overline{\psi}$ of original 2-component fields to a corresponding set of 1-component complex fields ϕ and F. (Here and in the following, $\overline{\psi}$ means $\overline{\psi}_\pm$.)

First, for each ω (and in the temporal gauge), go to a representation that diagonalizes the Hermitian operator $i\sigma^k D_k$:

$$\begin{aligned} \overline{\psi}\,(x) &= \sum_{\omega,s} a_{\omega,s}\overline{\psi}_s\left(\overrightarrow{x}\right)e^{-i\omega x^0} \\ i\,\sigma^k\left(\partial_k + iA^i_k t_i\right)\overline{\psi}_s\left(\overrightarrow{x}\right) &= P_s\overline{\psi}_s\left(\overrightarrow{x}\right)\ . \end{aligned}$$

This has the form

$$-\sigma^k P_k\overline{\psi}_s\left(\overrightarrow{x}\right) = P_s\overline{\psi}_s\left(\overrightarrow{x}\right)\ .$$

Since $\sigma^2\sigma^k\sigma^2 = -\left(\sigma^k\right)^*$, and P_k and P_s are real, there is another $\overline{\psi}_{-s}$ with the opposite eigenvalue:

$$-\sigma^k P_k\overline{\psi}_{-s}\left(\overrightarrow{x}\right) = -P_s\overline{\psi}_{-s}\left(\overrightarrow{x}\right)\ .$$

We will call the positive eigenvalue P_s and the negative eigenvalue $-P_s$, and will include both as a single term in the sums below.

Let us now return to the action for positive frequency states, with $\overline{\psi}$ representing $\overline{\psi}_+$:

$$
\begin{aligned}
S_+ &= \int d^4x\, \overline{\psi}^\dagger(x)\;\; i\,\sigma^\mu D_\mu\, \overline{\psi}(x) \\
&= \sum_{\omega,s} \left[a^*_{\omega,s}\,(\omega+P_s)\,a_{\omega,s} + a^*_{\omega,-s}\,(\omega-P_s)\,a_{\omega,-s}\right]\ .
\end{aligned}
$$

Letting

$$
\begin{aligned}
a_{\omega,-s} &= (\omega+P_s)^{1/2}\,\phi_{\omega,s} \\
a_{\omega,s} &= (\omega+P_s)^{-1/2}\,F_{\omega,s}
\end{aligned}
$$

we obtain

$$
\begin{aligned}
S_+ &= \sum_{\omega,s} \left[\phi^*_{\omega,s}\left(\omega^2-P_s^2\right)\phi_{\omega,s} + F^*_{\omega,s}F_{\omega,s}\right] \\
&= \int d^4x\;\left[\phi^\dagger(x)\,\eta^{\mu\nu}D_\mu D_v\phi(x) + F^\dagger(x)\,F(x)\right]\ .
\end{aligned}
$$

Here ϕ and F are positive frequency fields:

$$\begin{aligned}
\phi\left(x\right) &= \sum_{\omega,s} \phi_{\omega,s}\chi_s\left(\overrightarrow{x}\right)e^{-i\omega x^0} \\
F\left(x\right) &= \sum_{\omega,s} F_{\omega,s}\chi_s\left(\overrightarrow{x}\right)e^{-i\omega x^0} \\
\delta^{kl}D_kD_l\chi_s\left(\overrightarrow{x}\right) &= P_s^2\chi_s\left(\overrightarrow{x}\right)
\end{aligned}$$

with

$$\begin{aligned}
\phi_{\omega,s} &= \int d^4x\,\chi_s^\dagger\left(\overrightarrow{x}\right)e^{i\omega x^0}\phi\left(x\right) \\
F_{\omega,s} &= \int d^4x\,\chi_s^\dagger\left(\overrightarrow{x}\right)e^{i\omega x^0}\phi\left(x\right) \quad .
\end{aligned}$$

For negative frequency states, we have, with $\overline{\psi}$ now representing $\overline{\psi}_-$,

$$
\begin{aligned}
S_- &= -\int d^4x \, \overline{\psi}^\dagger(x) \;\; i\,\sigma^\mu D_\mu \overline{\psi}(x) \\
&= -\sum_{\omega,s} \left[a^*_{\omega,-s} \left(-|\omega| - P_{\omega,s} \right) a_{\omega,-s} + a^*_{\omega,s} \left(-|\omega| + P_s \right) a_{\omega,s} \right]
\end{aligned}
$$

so in this case define

$$
a_{\omega,s} = \left(|\omega| + P_s \right)^{1/2} \phi_{\omega,s} \quad , \quad a_{\omega,-s} = \left(|\omega| + P_s \right)^{-1/2} F_{\omega,s} \; .
$$

Then we again obtain the form

$$
\begin{aligned}
S_- &= \sum_{\omega,s} \left[\phi^*_{\omega,s} \left(\omega^2 - P_s^2 \right) \phi_{\omega,s} + F^*_{\omega,s} F_{\omega,s} \right] \\
&= \int d^4x \, \left[\phi^\dagger(x) \, \eta^{\mu\nu} D_\mu D_v \phi(x) + F^\dagger(x) \, F(x) \right]
\end{aligned}
$$

but in this case ϕ and F are negative frequency fields:

$$
\phi(x) = \sum_{\omega,s} \phi_{\omega,s} \chi_s(\overrightarrow{x}) \, e^{i|\omega| x^0} \quad , \qquad F(x) = \sum_{\omega,s} F_{\omega,s} \chi_s(\overrightarrow{x}) \, e^{i|\omega| x^0} \; .
$$

The total bosonic action, including both positive and negative frequencies, then still retains exactly this form:

$$S_b = S_+ + S_- = \int d^4x \, \left[\phi^\dagger(x)\,\eta^{\mu\nu}D_\mu D_\nu \phi(x) + F^\dagger(x)\,F(x)\right] \; .$$

With the fermionic action left untouched, and after an integration by parts, we obtain the standard supersymmetric action

$$S_{fb} = \int d^4x \, \left[\psi_f^\dagger \, i\sigma^\mu D_\mu \psi_f - \eta^{\mu\nu} D_\mu \phi^*(x)\, D_\nu \phi(x) + F^*(x)\, F(x)\right]$$

where the fields ϕ, F, and ψ_f respectively consist of 1-component complex scalar bosonic fields ϕ_p, 1-component complex scalar auxiliary fields F_p, and 2-component spin 1/2 fermionic fields ψ_p.

Reverting back to a general coordinate system, we have

$$S_{fb} = \sum_p \int d^4x\, e\, \left[\overline{\psi}_p^{\dagger}\, i e_\alpha^\mu\, \sigma^\alpha \widetilde{D}_\mu \overline{\psi}_p - g^{\mu\nu} \widetilde{D}_\mu \overline{\phi}_p^{*}(x)\, \widetilde{D}_\mu \overline{\phi}_p(x) + F_p^{*}(x)\, F_p(x) \right]$$

with

$$\begin{aligned} \overline{\psi}_p &= e^{-1/2}\, \psi_p \\ \overline{\phi}_p &= e^{-1/2}\, \phi_p\ . \end{aligned}$$

Again, $e = \det e_\mu^\alpha = \left(-\det g_{\mu\nu}\right)^{1/2}$, where e_μ^α is the vierbein, $g_{\mu\nu} = e_\mu^\alpha e_\nu^\alpha$ is the metric tensor, and

$$\widetilde{D}_\mu = D_\mu + e^{-1/2} \partial_\mu e^{1/2} \quad , \quad D_\mu = \partial_\mu + i A_\mu^i t_i\ .$$

(One can carry through the above treatment in a general coordinate system to get this expression for S_{fb}, with no essential differences.) The coupling of matter to gravity is thus very nearly the same as in standard general relativity.

However, if S_{fb} is written in terms of the original fields ψ_p and ϕ_p, there is no factor of e. In other words, in the present theory the original action has the form

$$S_0 = \int d^4x \, \overline{\mathcal{L}}_0$$

whereas in standard physics it has the form

$$S_0 = \int d^4x \, e \, \overline{\mathcal{L}}_0 \ .$$

For an $\mathcal{L}$ corrsponding to a vacuum energy density, there is then no coupling to gravity in the present theory, and the usual cosmological constant vanishes.

On the other hand, there should be a "diamagnetic response" of vacuum fields to changes in both the gauge fields and vierbein, which results from a shifting of the energies of the vacuum states when fields are applied, just as the energies of the electrons in a metal are shifted by the application of a magnetic field. We posulate that this effect produces contributions to the action which are respectively consistent with the gauge symmetry and the general coordinate invariance which the present theory exhibits at low energy. The lowest order such contributions are, of course, the Maxwell-Yang-Mills and Einstein-Hilbert actions, plus a relatively weak cosmological constant arising from this same mechanism:

$$\begin{aligned}\mathcal{L}_g &= -\frac{1}{4} g_0^{-2} F^i_{\mu\nu} F^i_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma} \\ \mathcal{L}_G &= e\,\Lambda + \ell_P^{-2} e\; {}^{(4)}R\,.\end{aligned}$$

Here g_0 is the coupling constant for the fundamental gauge group (e.g.$SO(10)$), Λ is a constant, and $\ell_P^2 = 16\pi G$. These terms are analogous to the usual contributions to the free energy from Landau diamagnetism in a metal.

As pointed out in IJMP1997, the gauge and gravitational curvatures require that the order parameter contain a superposition of configurations containing topological defects (without which there could be no curvature). Here we do not attempt to discuss these defects in detail, but they are analogous to vortex lines in a superfluid.

The actions for gauginos and gravitinos are postulated to have a similar origin, as the vacuum responds to these fields.

Particle masses and Yukawa couplings are postulated to arise from supersymmetry breaking and radiative corrections.

There is clearly a lot of work remaining to be done – including actual predictions for experiment – but the theory is relatively close to real-world physics, and the following arise as emergent properties from a fundamental statistical picture:

- standard physics
- $SO(N)$ fundamental gauge theory
- supersymmetry
- gravitational metric, with the form $(-,+,+,+)$
- correct coupling of matter fields to gravity
- vanishing of usual cosmological constant
- spacetime and fields

Criticisms and suggestions are welcomed!